

# Extracting information from commutative grids via zigzag persistent homology

Yasuaki Hiraoka<sup>1</sup>, Ken Nakashima<sup>2</sup>,  
Ippei Obayashi<sup>2</sup>, and Chenguang Xu<sup>1</sup>

<sup>1</sup>Kyoto University, <sup>2</sup>Okayama University

## Background:

## what is TDA (topological data analysis)?

Image Credit: Shawhin Talebi @ towardsdatascience.com

- Large amounts of data are being generated at an unprecedented speed.



Data



Shape



$$\begin{aligned} N &= 4 \\ E &= 3 \\ H_0 &= 1 \\ H_1 &= 0 \end{aligned}$$

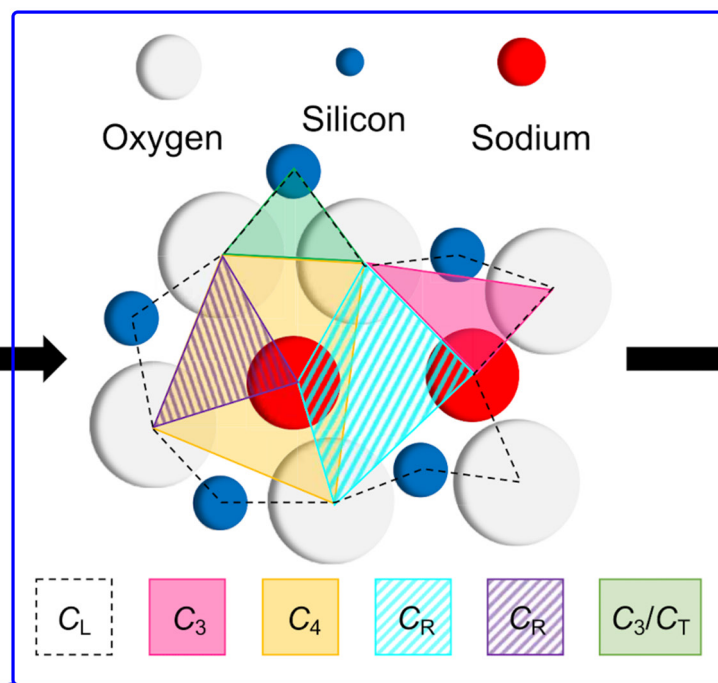
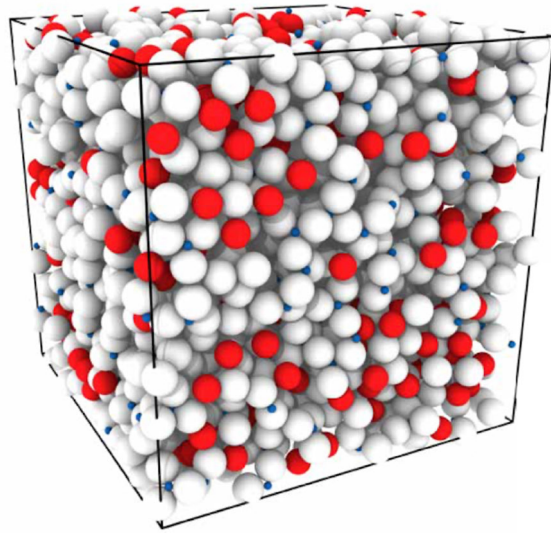
Topological Features



Analysis

- Topological data analysis (TDA) uses topological and geometric tools to infer topological features from high-dimensional and noisy data.
- Reveal structures in unstructured data as an algebraic summary.

## 20Na<sub>2</sub>O-80Si<sub>2</sub>O glass



## Persistence diagram

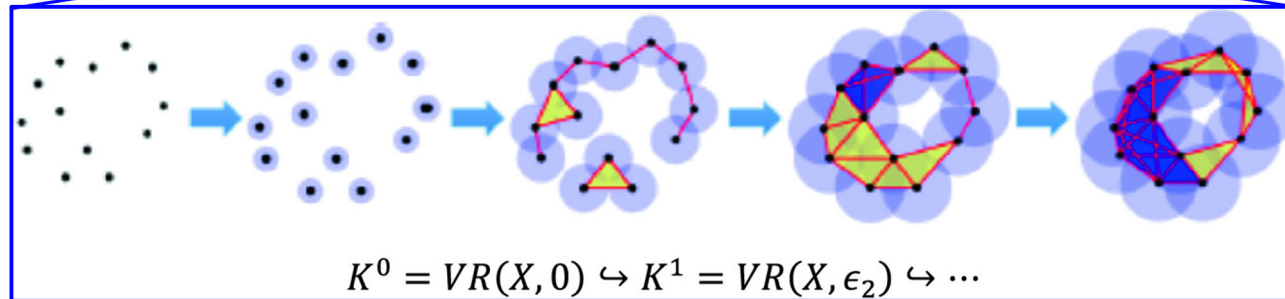
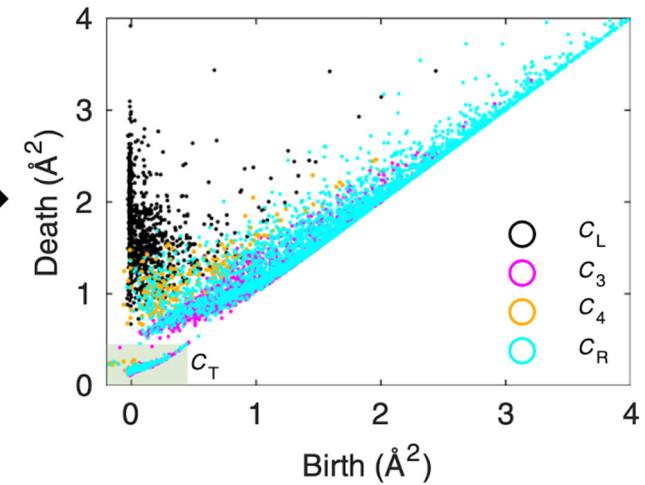


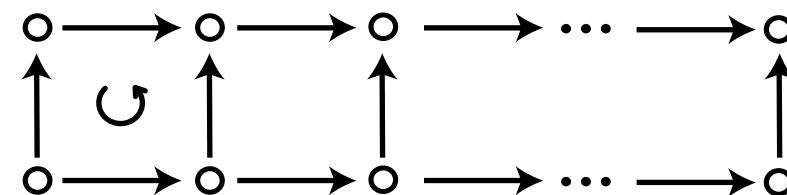
Image credits: Revealing hidden medium-range order in amorphous materials using topological data analysis, and Persistent-Homology-based Machine Learning and its Applications--A Survey.

- One-parameter **persistent homology** (PH) is the primary tool of TDA, and it provides a descriptor for order in disorder systems.
- A filtration of topological spaces can be built from a given point cloud.
- Then we compute the homology group of that filtration and record the results in the form of a **persistent diagram** (PD).

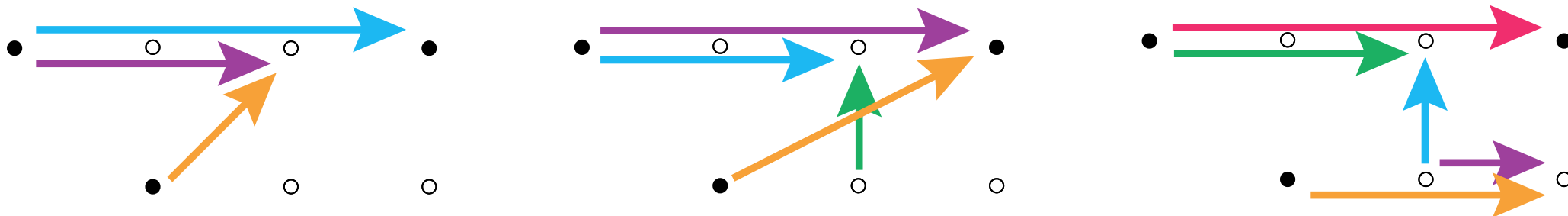
# Methods and main results

- A multi-parameter filtration of topological spaces will provide better versatility to incorporate real-world data.
- However, obtaining invariants in this setting is highly challenging and computationally expensive.
- We provide a framework for computing some invariants in them via tracing down different one-parameter tours and then linearly combine the results.

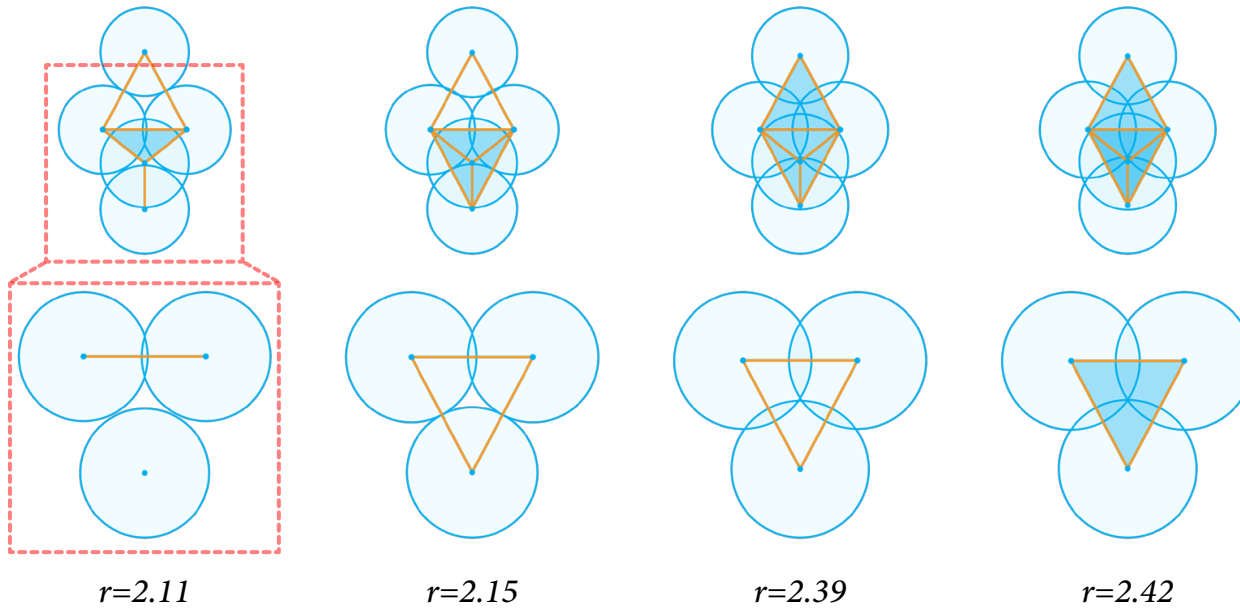
$$\delta_M^\xi(I) = \sum_{S \subseteq \text{Cov} I} (-1)^{\#S} c_M^\xi(\bigvee S)$$



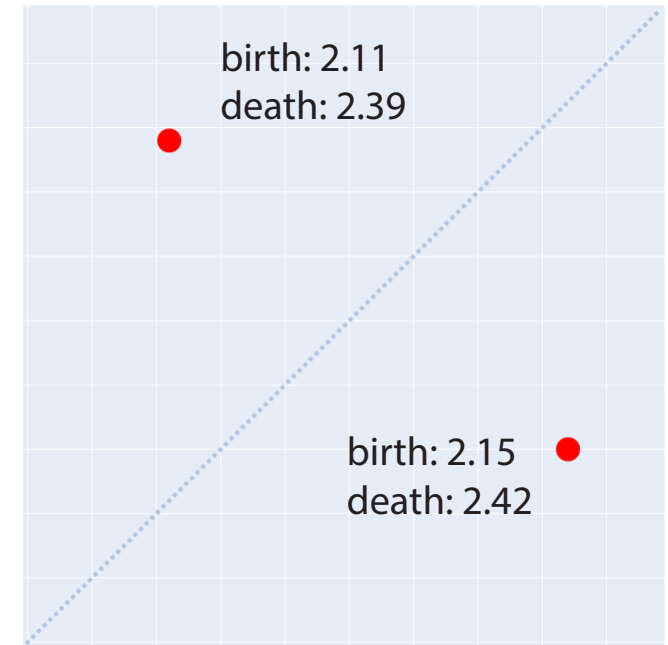
- We apply our framework to the ladder-shaped filtration illustrated.
- Finding new types of tours is essential in this process.



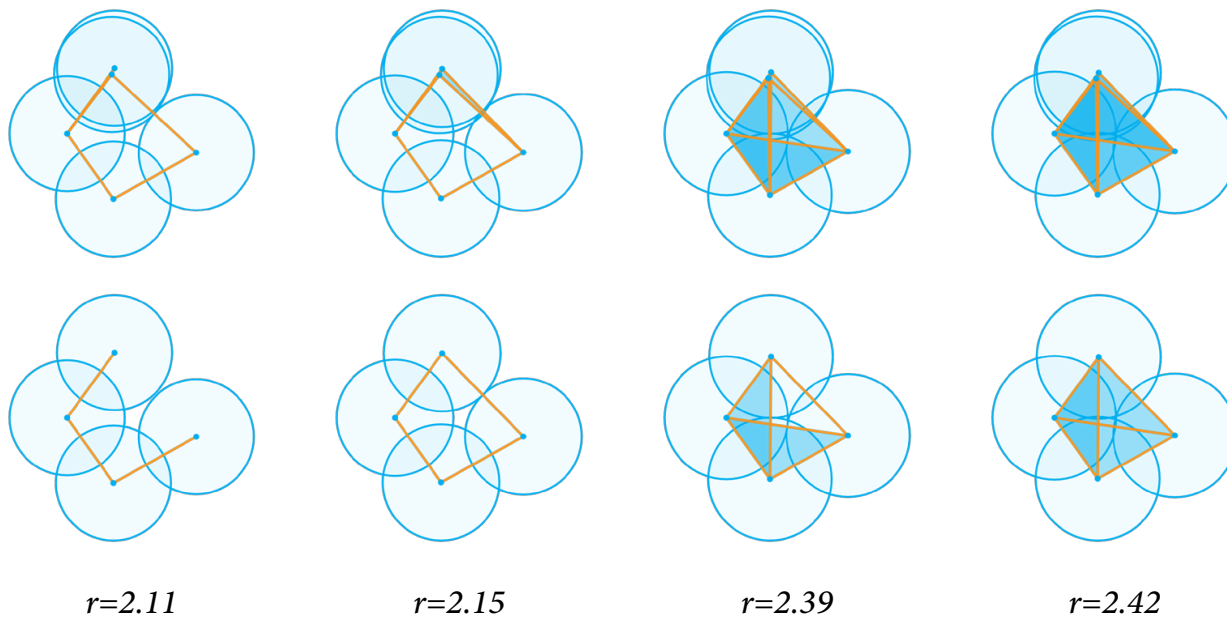
- Proposing a new invariant called ***connected persistence diagram (cPD)***.
- Provides a partial measurement of the vertical persistence



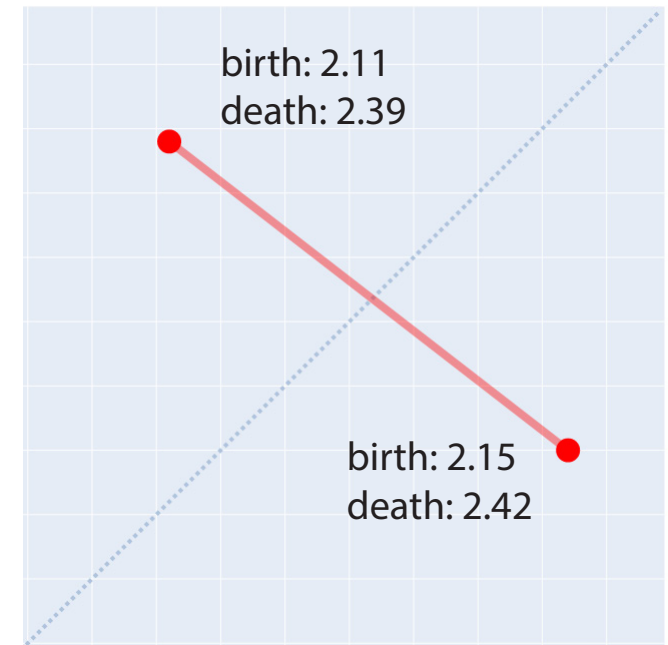
(a-1)



(a-2)



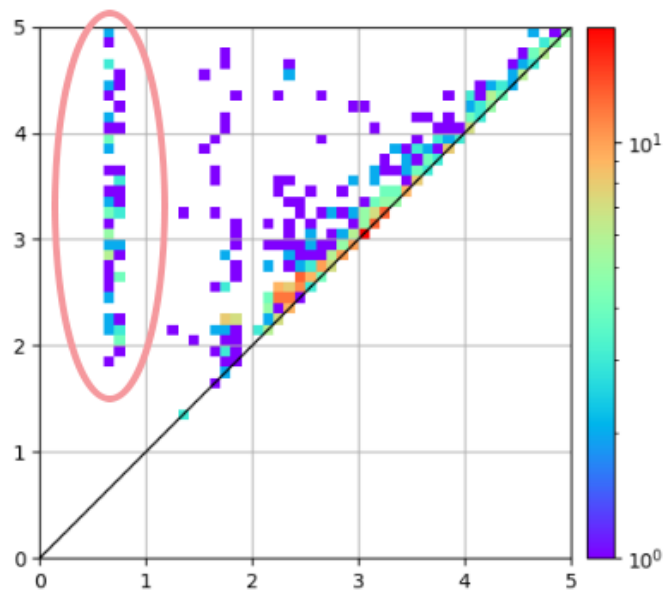
(b-1)



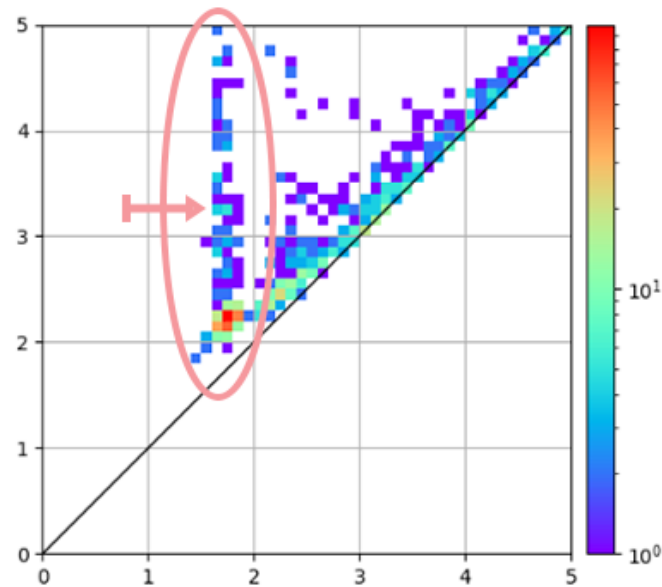
(b-2)

# An application: silica

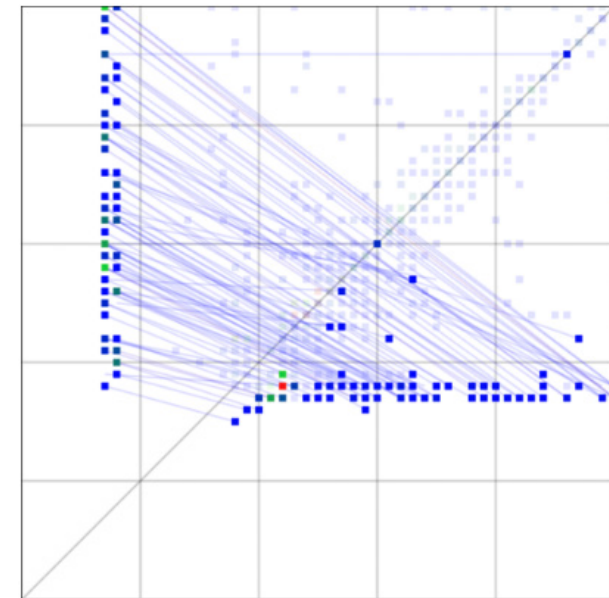
- Analyzing the atomic arrangement of silica using cPDs.
- See how the structure changes when all silicon atoms are removed while all oxygen atoms remain intact.



(a)



(b)



(c)

- The connecting lines in (c) show that the birth time of the main feature is delayed,
- **with its basic structure maintained.**