

The 41th Symposium on Transformation Groups

Abstract

November 13

Mikiya Masuda (Osaka City University)

Title: Cohomology of toric origami manifolds

Abstract: A toric origami manifold, where the closed 2-form on it is allowed to degenerate along a hypersurface, generalizes the notion of a symplectic toric manifold. As is well-known, there is a bijective correspondence between symplectic toric manifolds and so-called Delzant polytopes. Recently Cannas da Silva-Guillemin-Pires extended this result by showing that there is a bijective correspondence between toric origami manifolds and origami templates, where an origami template is a collection of Delzant polytopes satisfying a certain compatibility condition. So all the geometrical information on a toric origami manifold is encoded in the associated origami template but it is not completely understood how to read those geometrical information from the origami template. In this talk, we discuss the cohomology of a toric origami manifold, based on a joint work with Anton Ayzenberg, Seonjeong Park and Haozhi Zeng.

Masaharu Morimoto (Okayama University)

Title: Topological equivalence relations on representation spaces

Abstract: Let G be a finite group. There are interesting topological equivalence relations \sim_r in the family of real G -representation spaces of finite dimension, e.g. topological similarity \sim_t , homotopy equivalence \sim_h (in the sense of tom Dieck), Smith equivalence \sim_{sm} . Let $RO_r(G)$ be the subset of $RO(G)$ consisting of all $x = [V] - [W]$ such that $V \sim_r W$. We discuss classical results and new results on $RO_r(G)$.

Kohhei Yamaguchi (University of Electro-Communications)

Title: Atiyah-Jones type problem for the space of holomorphic maps on a certain toric variety

Abstract: After G. Segal proved the homology stability result for the space of the holomorphic maps from the Riemann sphere into a complex projective space in 1979, many topologists considered the similar problem for the space of holomorphic maps from the Riemann sphere into several projective varieties X (e.g. complex Grassmann manifold, flag manifold etc). In particular, M. Guest considered this type problem for the case of compact smooth toric variety in 1998. In this talk, the author considers this problem for the case of non-compact smooth toric variety. This talk is based on the joint work with A. Kozłowski (University of Warsaw).

November 14

Shintaro Kuroki (The University of Tokyo)

Title: On the extension of torus actions on GKM manifolds

Abstract: GKM manifold is a $2m$ -dimensional manifold with an (effective) n -dimensional torus action such that its orbit space which consisting of less than or equal to 1-dimensional orbits has the structure of a graph (called a GKM graph if we put a label on edges by tangential representations). By definition, n is less than or equal to m . In this talk, we define two combinatorial invariants on GKM graphs and solve the following two extension problems of torus actions on GKM manifolds: (1) If $n = m$, when does the torus action on a GKM manifold extend to an action of a compact connected Lie group? (2) When does the n -dimensional torus action on a GKM manifold extend to a torus action whose dimension is greater than n ? ((1) is a joint work with Prof. Mikiya Masuda)

Toshio Sumi (Kyushu University)

Title: Construction of gap modules

Abstract: Let G be a finite group. A gap G -module W is a finite dimensional real one satisfying two conditions:

1. $\dim W^L = 0$ for any subgroup L of G with prime power index
2. $\dim W^P > 2 \dim W^H$ for any subgroups P of prime power order and $H > P$ of G

By using this module, we can controll stably the gap among the dimensions of the fixed point sets by suitable subgroups of G and then change the G -action so that the fixed point set by G is reformed as we want, for example, the number of fixed points is decreased or increased. Thus it is important to decide which groups are gap groups. In this talk, we would like to introduce construction of gap modules.

Megumi Harada (McMaster University & Osaka City University)

Title: Newton-Okounkov bodies, representation theory, and Bott-Samelson varieties

Abstract: The theory of Newton-Okounkov bodies is a far-reaching generalization of the theory of toric varieties. In particular, it can associate to any complex projective variety X a convex body (which is a rational polytope in many cases) of dimension equal to the complex dimension of X ; in the case when X is a toric variety, the convex body is exactly the usual Newton polytope. Moreover, in a recent paper, Kaveh showed that the string polytopes in geometric representation theory are special cases of Newton-Okounkov bodies associated to flag varieties G/B . Hence the theory of Newton-Okounkov bodies is naturally related to many interesting questions in representation theory and Schubert calculus. The Bott-Samelson varieties give resolutions of singularities of Schubert varieties and are central in the study of the geometry of G/B . I will give an overview of this subject in relation to Newton-Okounkov bodies and discuss some recent and ongoing work, as well as some open questions.

Krzysztof Pawalowski (Adam Mickiewicz University)

Title: Transformation groups and Hsiangs' conviction after 46 years

Abstract: In 1968, Wu-Chung Hsiang and Wu-Yi Hsiang have expressed a conviction that reads as follows.

"Due to the existence of natural linear group actions on Euclidean spaces, disks and spheres, it is quite fair to say that they are the best testing spaces in the study of differentiable transformation groups... We share the prevailing conviction that the study of differentiable group actions on these best testing spaces is probably still the most important topic in transformation groups."

Based on Hsiangs' conviction, we survey a number of result in the theory of transformation groups that have been obtained during the past 46 years and we conjecture new results that should confirm the importance of understanding differentiable group actions on Euclidean spaces, disks and spheres, as well as projective spaces.

Marek Kaluba (Adam Mickiewicz University)

Title: Group actions on a class of 7-manifolds

Abstract: It is believed that a manifold "chosen at random" would have very few symmetries. In 1976 Raymond and Schultz asked for specific examples of simply-connected asymmetric manifolds, i.e. manifolds upon every compact group action is trivial. Examples of manifold models of $K(\pi, 1)$'s soon were found to be asymmetric by Borel and others, however the first simply connected manifold was proved to be (almost) asymmetric by Puppe nearly 20 years later. He showed an infinite family \mathcal{M}_{As} of simply connected 6-manifolds with no non-trivial orientation preserving group actions.

In this talk we will ask questions about symmetries of manifolds of the form $M \times S^n$ for $M \in \mathcal{M}_{As}$. We will prove that for $n = 2$ and G -cyclic, there are infinitely many exotic G -actions (i.e. actions not coming from the action on the sphere in the second factor). However in the case of $M \times S^1$ we will show that free actions of finite odd order cyclic groups or the circle are always standard (by rotation on the S^1 -factor). This is a joint research with Zbigniew Błaszczyk.

Norihiko Minami (Nagoya Institute of Technology)

Title: TBA

Abstract:

November 15

Tatsuhiko Yagasaki (Kyoto Institute of Technology)

Title: Homeomorphism groups of non-compact surfaces endowed with the Whitney topology

Abstract: We study topological type of the homeomorphism group $H(M)$ of any non-compact connected surface M endowed with the Whitney topology and show that the identity connected component $H_0(M)$ of $H(M)$ is homeomorphic to the product of l_2 and R^∞ . A survey of the following paper:

- T.Banakh, K.Mine, K.Sakai, T.Yagasaki, *On homeomorphism groups of non-compact surfaces, endowed with the Whitney topology*, Topology Appl., 164 (2014) 170–181.

Ikumitsu Nagasaki (Kyoto Prefectural Univ. of Medicine)

Title: On bi-isovariantly equivalent representations

Abstract: As an application of the isovariant Borsuk-Ulam theorem, we show that if there exist isovariant maps bidirectionally between representations V and W of a compact Lie group G , then the dimension functions of V and W coincide. If G is abelian, then the converse also holds. On the other hand, when G is a dihedral group, we give an example such that the converse does not hold for certain real 2-dimensional representations. (These are a part of joint work with F. Ushitaki.)

Taras Panov (Moscow State University)

Title: On the rational formality of toric spaces and polyhedral products

Abstract: Several important toric spaces, such as toric and quasitoric manifolds, moment-angle complexes and their partial quotients admit homotopy theoretical decomposition into homotopy colimits of diagrams over the face category $cat(K)$ of a simplicial complex K . A general construction of this sort is the polyhedral power X^K of a space X .

We establish formality (in the sense of rational homotopy theory) of the polyhedral power X^K with formal X , as well as formality of (quasi) toric manifolds and some torus manifolds. This contrasts the situation with moment-angle complexes $Z_K = (D^2, S^1)^K$, which are not formal in general.

Zhi Lu (Fudan University)

Title: Equivariant unitary bordism and equivariant cohomology Chern numbers

Abstract: Let G be a torus. In this talk, using the universal toric genus and the Kronecker pairing of bordism and cobordism, we show that the integral equivariant cohomology Chern numbers completely determine the equivariant geometric unitary bordism classes of closed unitary G -manifolds, which gives an affirmative answer to the conjecture posed by Guillemin–Ginzburg–Karshon in [1, Remark H.5, §3, Appendix H]. As a further application, we also obtain a satisfactory solution of [1, Question (A), §1.1, Appendix H] on unitary Hamiltonian G -manifolds. In particular, our approach can also be applied to the study of $(\mathbb{Z}_2)^k$ -equivariant unoriented bordism, and without the use of Boardman map, it can still work out the classical result of tom Dieck [2], which states that the $(\mathbb{Z}_2)^k$ -equivariant unoriented bordism class of a smooth closed $(\mathbb{Z}_2)^k$ -manifold is determined by its $(\mathbb{Z}_2)^k$ -equivariant Stiefel–Whitney numbers. In addition, we also show the equivalence of integral equivariant cohomology Chern numbers and equivariant K-theoretic Chern numbers for determining the equivariant unitary bordism classes of closed unitary G -manifolds by using the developed equivariant Riemann–Roch relation of Atiyah–Hirzebruch type, which implies that, in a different way, we may induce another classical result of tom Dieck, saying that equivariant K-theoretic Chern numbers completely determine the equivariant geometric unitary bordism classes of closed unitary G -manifolds. This is a joint work with Wei Wang.

1. V. Guillemin, V. Ginzburg and Y. Karshon, *Moment maps, cobordisms, and Hamiltonian group actions*. Appendix J by Maxim Braverman. Mathematical Surveys and Monographs, **98**. American Mathematical Society, Providence, RI, 2002.
2. T. tom Dieck, *Characteristic numbers of G -manifolds. I*. Invent. Math. **13** (1971), 213–224.