Group Actions on a Class of 7-manifolds.

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$$G \times (M \times N) \longrightarrow M \times N$$
$$(g,(x,y)) \longmapsto \begin{bmatrix} \varphi(g) & 0 \\ 0 & \psi(g) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Where φ and ψ denote actions of G on maifolds M, N respectively.

Motivation

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Choose M with as *few symmetries* as possible.

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> *What is the minimal n (depending on M and G) such that there exist a non-product action of G on $M \times S^n$?*

Asymmetric manifolds

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Theorem

- *There exist an infinite family of simply connected, 6-dimensional smooth manifolds which do not admit any effective (even topological) action of any compact Lie group with possible* **exception** *of orientation reversing involutions.*

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Simply connected 6-dimensional manifolds with little symmetry (...)

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- *But if we are satisfied with just* **topological manifolds** *then there exists a similar family of* **non-smoothable** *ones which admit no involutions at all*
- *Existence of smooth simply connected manifolds with no involutions is still an open problem.*

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A G-action on $M \times N$ is a product action if and only if both projections $\pi_M \colon M \times N \to M$ and $\pi_N \colon M \times N \to N$ are G-equivariant maps.

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Let $G = S^1$ or $\mathbb{Z}/_p$. Suppose that for every G-action on M, M^F is connected and that there is an action on $M \times S^n$ with an H-isotropy set

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Proposition

Let M be a n-dimensional asymmetric manifold. There exist effective, non-product actions of G on $M \times S^2$.

Codimension 2 fixed point sets of G-actions

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Let X be a contractible, $(n+1)$-dimensional ($n \geq 3$) manifold with smooth boundary $\partial X = F$. Then there exist effective, smooth G-action on sphere S^{n+2} with the fixed-point set diffeomorphic to F.

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Every codimension 2 fixed point set S^1-action on a sphere comes from this construction, by result of W-Y. Hsiang

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- Choose a n-dimensional ($n \geq 3$) non-simply connected manifold F bounding a contractible manifold X.
- By the previous proposition there exists a smooth action of G on S^{n+2} with the fixed point set diffeomorphic to F and tangential G-module at F isomorphic to $V \oplus n\mathbf{1}_G$.

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- Form the connected sum

$$M \times S(V \oplus \mathbb{R}) \# S^{n+2} \cong M \times S^2.$$

(e.g. F may be a smooth homology sphere)

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- Since all actions on S^2 are linear, a product action on $M \times S^2$ would have the fixed point set either
  - empty (fixed-point-free action on S^2), or
  - diffeomorphic to $M \sqcup M$ (2-fixed-points action on S^2), or
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- Observe that the fixed point set of the action constructed on $M \times S^2$ consists of two components

$$M \sqcup M \# F$$

with non-isomorphic fundamental groups.

$\square$

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We strongly believe that the following is also true:

Theorem? (Work in progress)

All free $\mathbb{Z}/_p$-actions on $M \times S^1$ are equivalent to a product action ($p \neq 2$).

Product action = id $\times \exp\left(\frac{2\pi i}{p}\right)$

Proof: (free S^1-actions)

We use the fact that a free S^1-action on $M \times S^1$ yields a fibre bundle over the orbit space $X \stackrel{\text{def.}}{=} M \times_G S^1$:

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Every such bundle has a classifying map

$$X \xrightarrow{c(\xi)} BS^1$$

We want to use the map to compare fibre bundles.

Proof: (continued)

All such S^1-bundles are determined by their first Chern class

$$c_1(\xi) = c(\xi)^*(x),$$

where x is the generator of $H^2(BS^1, \mathbb{Z})$.

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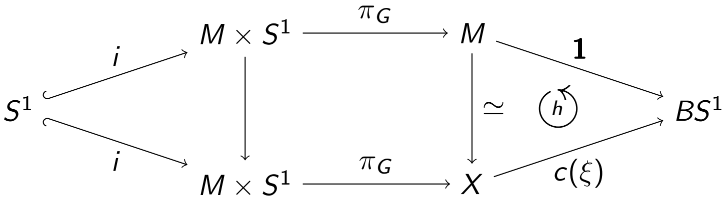
Assume so for now.

Proof: (continued)

Then we have a commuting diagram:



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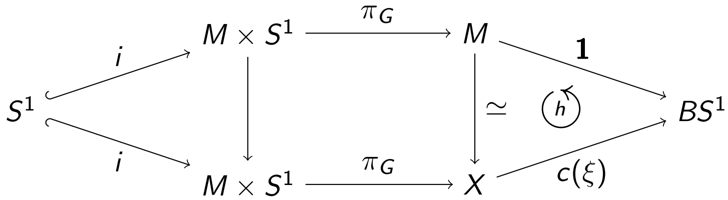

So we know that over (a manifold) X the trivial S^1-bundle satisfies

$$M \times S^1 \cong X \times S^1.$$

Proof: (continued)

Observe that this gives us just a homotopy equivalence

$$M \xrightarrow{\simeq} X.$$

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Exercise (in *h*-cobordism)

Improve this to a diffeomorphism.

Solution:

We already have a diffeomorphism $M \times S^1 \to X \times S^1$. Lift it to the $\mathbb{Z}$-cover

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The image $\varphi(M \times \{0\})$ belongs to $X \times (0, 2)$ and separates $X \times \mathbb{R}$ into two components. Choose one of them and intersect it with $\big(X \times (\pm\infty, a]\big)$.

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This is a non-empty, connected manifold with boundary

$$\partial W \cong N \sqcup \varphi(M).$$

Moreover the inclusions $N \hookrightarrow W$ and $\varphi(M) \hookrightarrow W$ are homotopy equivalences. Since $\pi_1(M) = 0$ we obtain a diffeomorphism $M \to X$ by the h-cobordism theorem.

Proof: (end of)

So M and X are diffeomorphic, and the diffeomorphism gives us desired equivalence of actions.

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Proof of this fact relays on:

Fact: Multiplication by $c_1(\xi)$ can be identified with a differential on the second page of the Leray-Serre spectral sequence.

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Then we use cohomological properties of M to prove that $c_1(\xi) = 0$.

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Assume that $\pi_1(X)$ acts trivially on $H^*(S^1)$. Then we have the following spectral sequence

$$E_2^{p,q} = H^p(X, H^q(S^1, \mathbb{Z})) \Rightarrow H^{p+q}(M \times S^1, \mathbb{Z}).$$

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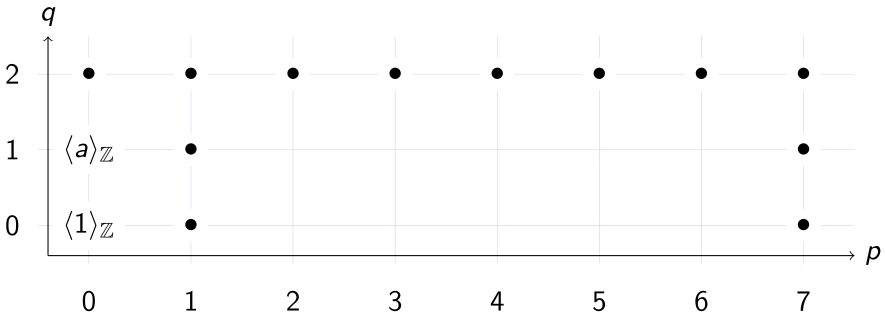

q
p
2
1
0
⟨a⟩ℤ
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0
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4
5
6
7

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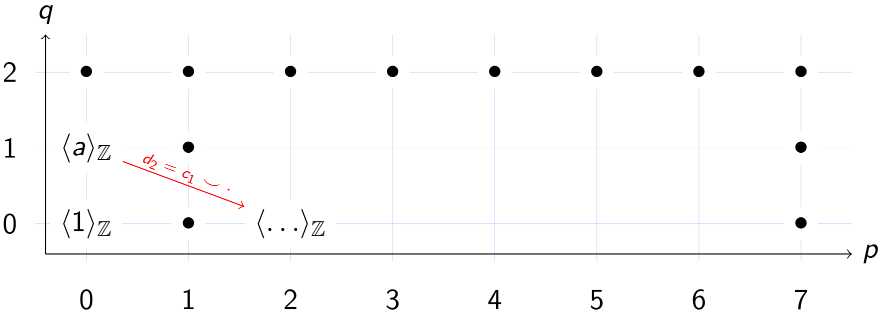

- $d_2 \colon E_2^{0,1} \to E_2^{2,0}$ is multiplication by $c_1(\xi)$
- Set $d_2(a) = c \neq 0$. We claim that $c \in \mathbb{Z}/_k$ is a generator.

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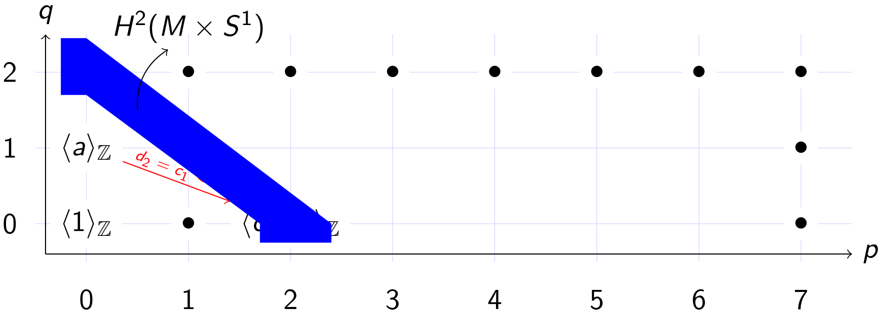

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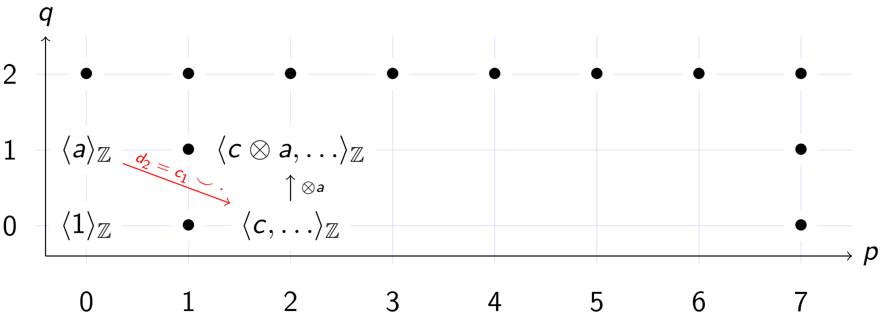

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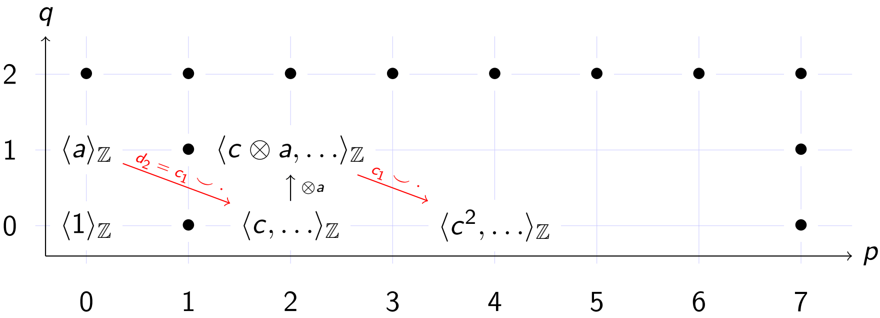

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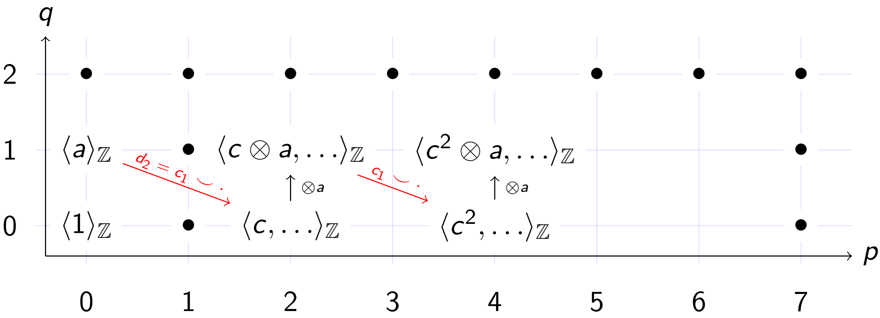

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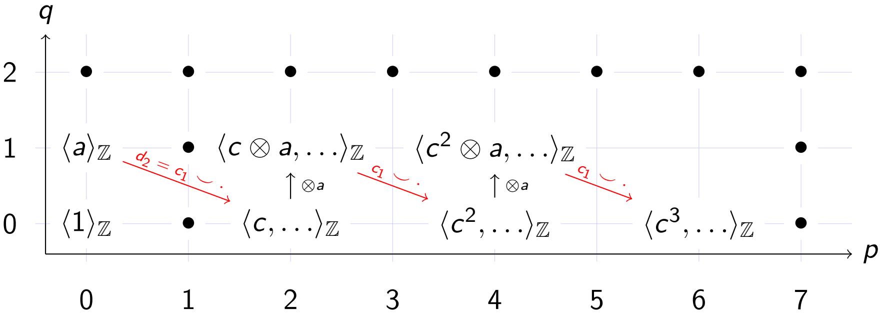

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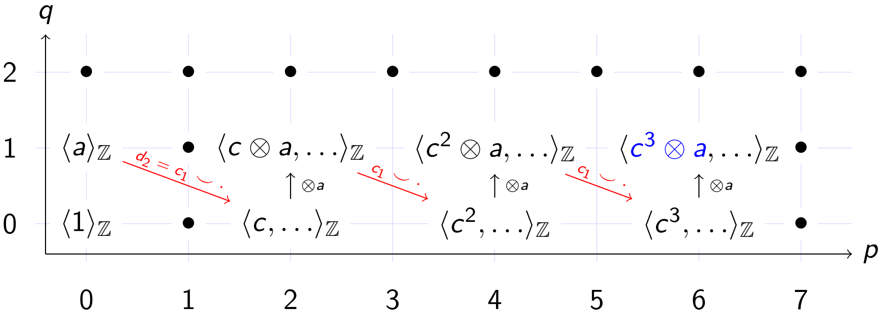

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- $c^3 \otimes a$ survives to E_∞ and hence to $H^7(M \times S^1)$.

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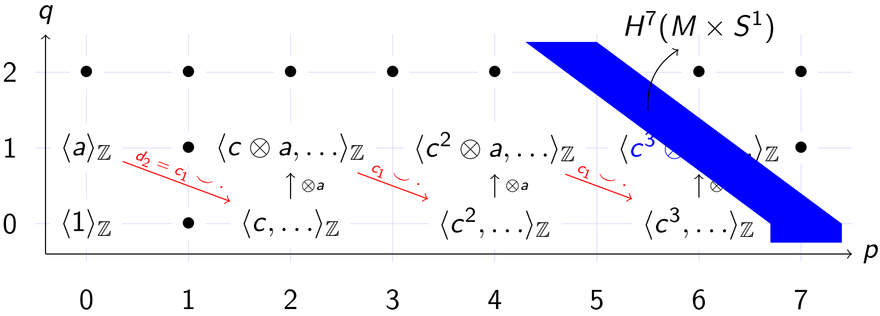

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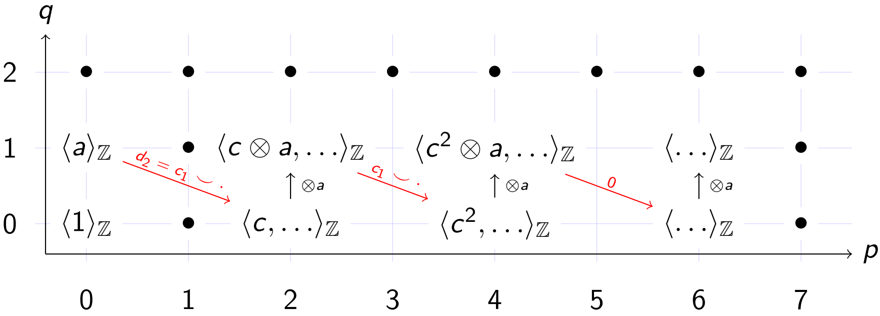

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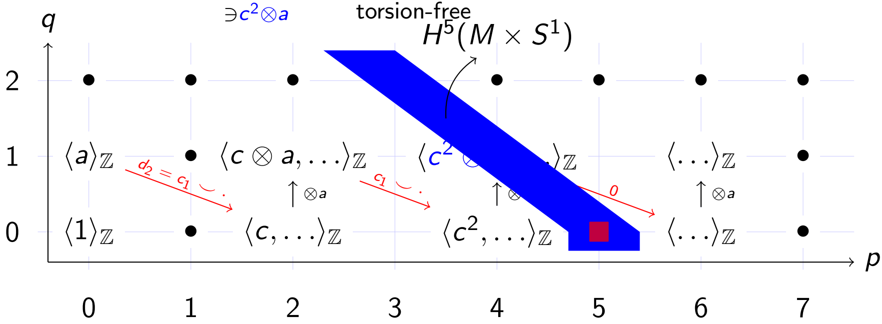
- But $H^7(M \times S^1) = \mathbb{Z}$, so $d_2(c^2 \otimes a) = c^3 = 0$.
- Now $c^2 \otimes a$ survives to E_∞, so we have an extension

by multiplicative properties

$$0 \to \underbrace{E_2^{4,1}}_{\ni c^2 \otimes a} \hookrightarrow \underbrace{H^5(M \times S^1)}_{\text{torsion-free}} \to \blacksquare \to 0.$$



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The proof above suggests, that the fact is more general, i.e. it holds for all manifolds M with torsion-free cohomology in even degrees.

Further perspective

	$\mathbb{Z}/_p$ groups	circle
non-free actions		
free actions		

Is every $\langle * \rangle$ action of G on $M \times S^2$ product?

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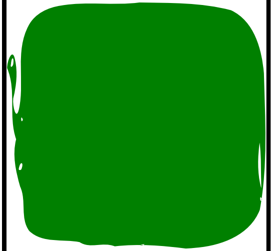
free actions		

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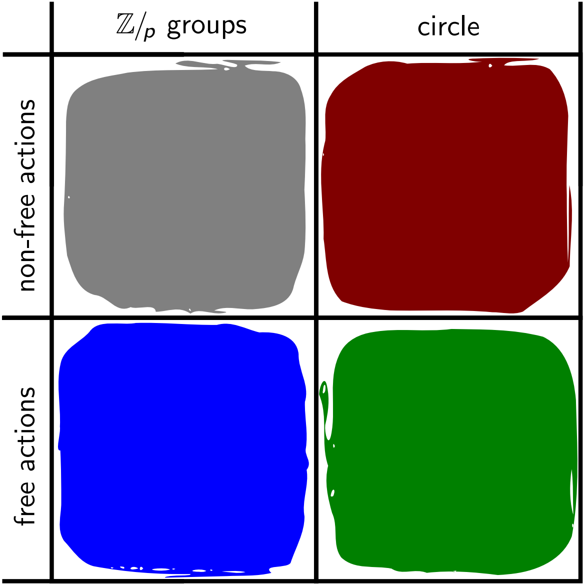

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We believe that for free $G = \mathbb{Z}/_p$-actions the homotopy type of the orbit space is the invariant.

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A similar results on free $\mathbb{Z}/_p$-actions on $S^n \times S^1$ was recently obtained by Q.Khan (for p an odd prime) and B.Jahren&S.Kwasik (for p an even prime).

Conjectures and problems

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Problem (for a decent-lunch-price)

What are algebraic or geometric (computable!) invariants that will allow us to recognize a product action?

ありがとう