Origin of the Experiment of Impact with Pendulums

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It is well known that Galileo's experiments on the pendulum resulted in discoveries which played fundamental roles in the formation of classical mechanics. Galileo discovered the isochronism of the pendulum, which he later employed for the measurement of time; by comparing the oscillations of two pendulums of the same length having bobs made of different matters, he also established that the acceleration of falling bodies was always the same regardless of their density, size and shape; finally from an experiment which showed that the pendulum ascended to the same height as the starting point of descent, even when a part of the thread of the pendulum was fixed by a protruding nail during its oscillation, he inferred that the velocities acquired by falling bodies depend only on the vertical height of their fall. It might well be said that the Galilean mechanics would not have emerged without experiments on the pendulum. It is therefore quite natural that the great significance of the experiment on the pendulum for the Galilean mechanics has been repeatedly stressed and that the historic origin of the experiment on the pendulum has been eagerly studied.

It seems, however, that the origin of the experiment of impact with pendulums has scarcely studied, though it also played an important role in the formation of classical mechanics.[1] The historical importance of this experiment will easily be understood, if we consider that it was the only experiment in the seventeenth and eighteenth centuries that could produce an exact numerical result, and that it finally settled the long controversy about the theory of impact which had begun immediately after the publication of Descartes' *Principia Philosophiae* (1644).

Among the books of that time in which the experiment of impact with pendulums is treated in some detail, Newton's *Principia* is best known. In the scholium following the second preliminary section 'Axioms, or Laws of Motion' of the *Principia*, Newton describes an experiment with two pendulums to study

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[1] For example, R. Dugas makes any mention of the origin of the experiment of impact with pendulums neither in his *Histoire de la Mecanique*, Neuchatel, 1950, nor in his *Mecanique au XVIIe Siècle*, Neuchatel, 1954.

the law of impact.[2] Let two spherical bodies A and B be suspended by parallel and equal strings AC and BD from centers of oscillation C and D (Fig. 1). The arcs AE and BF are quadrants having their centers at C and D respectively. The bodies A and B are brought to arbitrary positions H and I, then they are released gently so that they may collide each other in the lowest position. After impact the bodies A and B are supposed to return respectively to the positions G and K. Making use of Galileo's law of falling bodies, it is easily shown that the velocities of the bodies A and B before impact are proportional to the cords AH and BI, and those after impact to the cords AG and BK. By measuring the lengths of these cords, therefore, we obtain the velocities of the two bodies after impact, which then could be compared with the values deduced from theory.

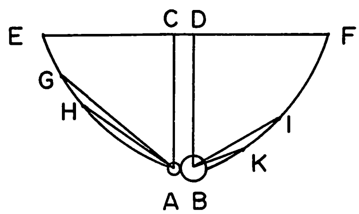

Fig. 1.

In the same scholium Newton also mentions the rules of impact presented to the Royal Society by Wallis, Wren and Huygens.

> Sir *Christopher Wren*, Dr. *Wallis* and Mr. *Huygens*, the greatest geometers of our times, did severally determine the rules of the impact and reflection of hard bodies, and about the same time communicated their discoveries to the *Royal Society*, exactly agreeing among themselves as to these rules. Dr. *Wallis*, indeed, was somewhat earlier in the publication, then followed Sir *Christopher Wren*, and lastly, Mr. *Huygens*. But Sir *Christopher Wren* confirmed the truth of the thing before the *Royal Society* by the experiment on pendulums, which *M. Mariotte* soon after thought fit to explain in a treatise [*Traité de la Percussion ou du Choc des Corps* (1673)] upon that subject.[3]

This passage gives us the impression that Wren was the first to make the experiment of impact with pendulums in order to verify his own rules of impact which were presented to the Royal Society in 1668, and that the experiments which Mariotte described in his *Traité de la Percussion ou du Choc des Corps* were merely imitation of Wren's experiment.

Henry Pemberton, who was the editor of the third edition of the *Principia*,

[2] Isaac Newton, *Mathematical Principles of Natural Philosophy*, Motte-Cajori version, Berkely and Los Angles, 1962, pp. 22sq.

[3] *Ibid.*, p. 22.

gives a similar account in his *View of Sir Isaac Newton's Philosophy*. He writes; "We may now proceed to those experiments upon the percussion of bodies, which I observed above might be made with pendulums. This expedient for examining the effect of percussion was *first proposed by our late great architect Sir Christopher Wren*."[4]

In the subsequent passage he describes in detail "this expedient," which is essentially the same as that described in the *Principia*. This example shows that, already in the second quarter of the eighteenth century, it became to be believed that Wren had been the first to make the experiment of impact with pendulums. This view seems to have been accepted until today by historians of science without being questioned.[5] While I was investigating the development of mechanics in the seventeenth century, I, however, found that the problem when and who had made the first experiment of impact with pendulums was not so simple as had appeared before. In the following I shall set down some facts which have been revealed by research of documentary material.

In the first place, there is reference to the pendulum experiment in Th. Birch's *History of the Royal Society of London*.[6] According to Birch, in October of 1666, two years before Wren's presentation of his own rules of impact to the Royal Society, an experiment of impact of two wooden globes suspended as pendulums was made. In the next week a similar experiment with three suspended globes was made. Regrettably, Birch does not mention who proposed these experiments. They, however, cannot be concluded to be the first experiments of this kind.

Henry Oldenburg, the first secretary of the Royal Society, answering an inquiry from Benedecti de Spinoza about Huygens' experiments of impact made in London, wrote on December 8, 1665;

> I was not present when Mr. Huygens performed the experiments confirming his hypothesis here in London. I have learned since then that among other things a ball weighing one pound was suspended as a pendulum; when it was released from angle of forty degrees it struck another ball weighing half a pound; Huygens, after making a little algebraic calculation, had predicted the effect. This occurred exactly as he predicted. One notable man, who had proposed many such experiments which Huygens is said to have resolved, is not here. As soon as I can meet this man who is

[4] Henry Pemberton, *A View of Sir Isaac Newton's Philosophy*, London, 1728, p. 98 (Italics mine).

[5] On this point, see, for example, Ernst Mach, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt*, Leipzig, 1933 (the ninth ed.), p. 317, and P. G. Tait, "Note on a Singular Passage in the *Principia*," *Proceedings of the Royal Society of Edingburgh*, vol. 13 (1886), pp. 72–78.

[6] Thomas Birch, *The History of the Royal Society of London* (a facsimile ed. 1968, New York and London), pp. 116–117.

now away I will perhaps explain this matter more fully and more exactly.[7]

Oldenburg seems not to have written another letter to Spinoza explaining Huygens' experiments made in London "more fully and more exactly." We however can learn the details of the experiments from Huygens' own description of them. In his *Journal de Voyage*, he writes; "23 [April 23, 1661]. I dined in my room. After dinner in my room assembled Mr. Moray, Mil. Brouncker, Sir Neile, Dr. Wallis, Mr. Rooke, Mr. Wren and Dr. Goddard. We talked about how to make lenses. I told them my method. I solved all the cases which they had proposed to me concerning the impact of spheres."[8]

This shows that experiments of impact with pendulums were already made in April of 1661. Moreover, judging from the quoted passages by Oldenburg and Huygens, it is inferred that someone of the seven English scientists mentioned above by Huygens proposed these experiments to him. In fact, from a letter which Moray wrote to Oldenburg on October 10, 1665, replying to his inquiry, it is concluded that Wren proposed these experiments, and that he and Rooke had actually carried out some pendulum experiments before their conversation with Huygens. Moray writes;

> The story of what Mr. Hugens says was thus, as both Dr. Wallis and I do well remember. When Mr. Hugens came out first over wee were with him at his lodging at the end of New Street in Convent Garden, where hee told us amongst other things of what he had done in the business of motion, as hee hath done to your friend of late. At that time Mr. Rook and Dr. Wren had made diverse experiments with balls of wood and other stuff hanging by threads whereof you may remember to have seen some, upon Mr. Hugens undertaking to solve all questions of motion according to his rule Dr. Wren did propose some which he had tryed by experiments, and Mr. Hugens did in a very short space solve them so as it was concluded by all the solution did agree with the experiments that have been made.[9]

What, then, should we think of Huygens, who established his own correct theory of perfectly elastic impact in the years 1652 to 1654, a few years before he visited London? Should we conclude that he had never hit on making such an experiment before it was proposed to him by English scientists? The answer is "no." There is, indeed, evidence in favor of Huygens. First, in one of his earliest manuscripts concerning the theory of impact, which were undoubtedly

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[7] Hall & Hall, eds., *The Correspondence of Henry Oldenburg*, vol. 2, Madison, Milwaukee and London, 1966, p. 636. There are some letters in existence which Moray and Wallis wrote to Oldenburg about what they had observed when these experiments were made. See Hall & Hall, eds., *op. cit*. There is no doubt that the information which Oldenburg communicated to Spinoza, had been obtained from Moray.

[8] *Oeuvres Complètes de Christiaan Huygens* (these books will be cited hereafter as O. H.), vol. 22, La Haye, 1950, p. 573.

[9] Hall & Hall, eds., *op. cit*., pp. 561-562.

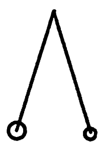

Fig. 2.

written in 1652, we find a figure as Fig. 2.[10] This suggests to us the possibility that he had the idea of making an experiment of impact with pendulums. Secondly, a similar figure is described in his manuscript of 1654.[11] Huygens also refers there to bobs made of ivory. Ivory is known as the most suitable material to make the experiment of perfectly elastic impact with. Huygens must have recognized this remarkable property of ivory in 1654. Lastly, he writes in his manuscript of 1656[12] that his theory of impact has been experimentally verified as exactly as theorems of mechanics. It will be reasonable to assume that his confidence in his theory of impact resulted from the success of experiments with pendulums having ivory bobs. If this view is not wrong, Huygens, as early as in 1656, must have known how to calculate the velocities of two suspended globes before and after impact. The fact that, when in 1661 Huygens was proposed the experiment of impact with pendulums, he could correctly predict the effect at once "after making a little algebraic calculation," can, I believe, be properly explained only when we accept the conclusion above.

As for Wren and other English scientists, it is evident from the above quoted letter of Moray that they devised the experiment of impact with pendulums independently of Huygens. But, according to Huygens,[13] they had not established the theory of impact as late as in April of 1661. Moreover, it is doubtful that they had known the method of calculating the velocities of two suspended globes before and after impact. For, though it was necessary to use the Galilean law of falling bodies in order to calculate the velocities in question, the English translation of Galileo's *Dialogo* by Th. Salusbery, which is said to have played a great role for the prevalence of Galileo's ideas of mechanics in England, had not yet been published. Moreover, there is some doubt about how eagerly they studied Galileo's *Discorsi*, and how familiar they were with concepts and theorems of Galileo's mechanics.[15] In contrast to this Huygens had been making a

[10] See a facsimile of this manuscript which is printed after the index of O. H., vol. 16, La Haye, 1929.

[11] O. H., vol. 16, La Haye, 1929, p. 107.

[12] O. H., vol. 16, pp. 139–140.

[13] O. H., vol. 6, La Haye, 1895, p. 383.

[14] The date of 'ad Lectorem' of this book is November 20, 1661.

[15] In March of 1662 William V. Brouncker, learning that Huygens had demonstrated the isochronism of the cycloidal pendulum, sent to Huygens a demonstration of it which he found by himself. It is interesting that Brouncker makes no reference to Galileo, but only to Stevin in his demonstration (see O. H., vol. 4, La Haye, 1891, pp. 88–91).

deep study of the *Discorsi* since 1646. Considering such a situation in England, we may surmise that it was by the wonderful agreement of Huygens' prediction with experimental result that Wallis and Wren were induced to study the theory of impact so eagerly that they could find their own rules of impact.

I have so far considered the experiment of impact with pendulums by Huygens and the fellows of the Royal Society. I do not, however, intend to assert that there was no other scientist who discussed a similar experiment. In fact, Galileo discussed an experiment of impact with pendulums in his treatise[16] which was written with the intention to publish as the sixth day of the *Discorsi*. This discussion led him to the conclusion that, if a moving body collide with another body at rest, the former would set the latter in motion, however small the former may be compared with the latter. But Galileo failed to reach the fruitful idea of using the pendulum experiment in order to obtain the velocities of colliding bodies before and after impact. His failure, however, may be said to have been inevitable, if we remember that he did not have the recognition that the velocities of colliding bodies before and after impact were, together with their masses, the fundamental factors. This recognition was first stated in Descartes' *Principia Philosophiae* which was published in 1644, two years after Galileo's death.

Any way, this treatise of Galileo had not been published before the beginning of the eighteenth century. It is evident, therefore, that the fellows of the Royal Society and Huygens were not directly influenced by the treatise. Yet there remains the possibility that they might have learned the idea of using pendulums to investigate the law of impact through Galileo's disciples such as Torricelli and Borelli. But, in so far as I have examined, Borelli nowhere refers to the idea in his voluminous *De vi percussionis* (1667). This fact suggests that it was within a very small private circle that Galileo's idea was, if any, communicated. It is therefore, at present, reasonable to say that Wren and other fellows of the Royal Society as well as Huygens reached the idea of the experiment of impact with pendulums independently of Galileo.

Finally a few words must be devoted to Mariotte. In the passage quoted above from the *Principia*, Newton says that Mariotte learned the experiment of impact with pendulums from Wren. This is probably wrong. It seems more reasonable to assume that he learned it from Huygens. For Huygens, making a reference to Mariotte's *Traité de la Percussion ou du Choc des Corps* in a manuscript[17] written in his last years, reproached him for plagiarizing the idea of the experiment from his lectures[18] on the theory of impact which were delivered in 1668 in the Academie des Sciences de Paris.

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[16] *Le Opere di Galileo Galilei*, Firenze, 1968, vol. 8, pp. 322-346.

[17] O. H., vol. 16, pp. 207-209.

[18] On these lectures, see, O. H., vol. 16, pp. 182-188.