

The Development of Nagaoka's Saturnian Atomic Model II (1904-05)

—Nagaoka's Theory of the Structure of Matter—

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1. Introduction

A research group, whose members are Dr. Kiyonobu Itakura, Mr. Tōsaku Kimura and myself, has worked on the biography of Hantaro Nagaoka since 1963. My participation in the biography is mainly connected with Nagaoka's activities in the period from 1903 to 1926, namely from his proposal of the Saturnian atomic model to his retirement from the University of Tokyo. I have published several papers on these activities.¹

Nagaoka proposed his Saturnian atomic model on December 5th 1903, at the monthly meeting of the Tokyo Physico-Mathematical Society and a year later his theory was published both in the *Proceedings of the Tokyo Physico-Mathematical Society*² and in the *Phil. Mag.*³ Nagaoka's model consisted of a number of negative electrons of equal mass, arranged uniformly in a circular ring, and a positively charged sphere of large mass, located at the center of the ring. In his paper Nagaoka tried to prove that if these mutually repelling electrons revolve with nearly the same velocity around the center, whose attracting force is sufficient, the whole system remains stable against minor disturbances.

The origin of Nagaoka's model has been discussed in my paper on Nagaoka's Saturnian Atomic Model (1903).⁴ The reason why he proposed this atomic model, rather than one similar to J. J. Thomson's has been also discussed by the use of Nagaoka's notes, correspondence, diaries, and review papers in Japanese. These materials had not been used before. It should be noticed that the uniqueness of

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¹ Eri Yagi, "On Nagaoka's Saturnian Atomic Model (1903)", *Jap. Stud. Hist. Sci.*, No. 3, 29-47, 1964; "The Development of Nagaoka's Saturnian Atomic Model I —Dispersion of Light—", *ibid.*, No. 6, 19-25, 1967; Errata, *ibid.*, No. 8, 177, 1969; "The Development of Nagaoka's Saturnian Atomic Model", *Actes XII^e Congrès International D'Histoire Des Sciences Paris 1968*, 5, 117-119, 1971; For the contents of our book on Nagaoka's biography, which will be published soon in Japanese by the Asahi Newspaper Publisher, see "Research Group of the Committee for the Publication of Hantaro Nagaoka's Biography", *Jap. Stud. Hist. Sci.*, No. 10, 23-24, 1971.

² Hantaro Nagaoka, see Appendix I 1.

³ H. Nagaoka, see Appendix I 4.

⁴ E. Yagi, *Ref.* 1).

Nagaoka's model was based on his material view of electricity, in other words, his corpuscular theory of electricity since few experiments had been made on the disposition of positive charge in an atom. Having been influenced by L. Boltzmann, Nagaoka rejected inter-penetrability between the two kinds of electricity, namely the positive and negative charges. Thus in his model he arranged negative electrons outside the central positive charge. Therefore the essential difference between Nagaoka's and J. J. Thomson's models lies in their conflicting views of electricity, rather than in their different ways of explaining the experimental facts of their times; such as spectrum lines, the Zeeman effect, and radioactivity. In J. J. Thomson's model negative electrons revolve freely inside the positive sphere, as if they were revolving in ether or in a cloud. On the contrary in Nagaoka's model the positive sphere is a large material particle into which electrons cannot penetrate.

In order to understand the characteristics of Nagaoka's atomic model and his theory of the structure of matter in more detail, it might prove useful to analyze a series of his theoretical papers, published in 1904 and 1905.⁵ Nagaoka dealt with the problems of band spectra, dispersion of light, index of refraction, the mutual action of atoms and the virial of molecular forces, on the assumption that molecules consist of a number of his Saturnian atoms. A list of the papers is given in Appendix I.

2. Extension of Deslandres' Formula

On April 20th 1904 Nagaoka published a paper, "Extension of Deslandres' Formula for a Band Spectrum",⁶ which was reported on March 19th at the monthly meeting of the Tokyo Physico-Mathematical society. As I have pointed out, Nagaoka's main purpose in constructing his model (1903) was to explain the observation that the Zeeman effect occurs only for the line spectrum while the band spectrum is not affected by an external magnetic field. (Note that the Zeeman effect on band spectrum had not been discovered at that time.) To explain this Nagaoka suggested that in an atom the vibrations, parallel to the plane of the circular orbit, correspond to line spectra, while the transverse vibrations, perpendicular to the plane, give band spectra. The frequency of the transverse vibration n is given in his first paper on his atomic model.

Nagaoka referred in his paper on the extension of Deslandres' formula to the bands of cyanogen to show a quantitative coincidence between his theory and observation. Nagaoka derived a formula for a type of band spectra:

$$n = a + bm^2 + cm^3 + dm^4 + em^5 + \dots \quad (m = 0, 1, 2, 3, \dots).$$

He then pointed out that this formula coincided with one of the empirical formulae

⁵ For the problem of dispersion of light, see E. Yagi, *Ref.* 1).

⁶ H. Nagaoka, *Tokyo Phys.-Math. Soc.*, [2], 2, 129-131, 1904.

used by H. Kayser and C. Runge as a modification of that of Deslandres. Further Nagaoka showed that from $m = 0$ to $m = 150$, the difference in wave lengths seldom exceeds 0.03×10^{-10} meters, which is within the limits of experimental errors.

A systematic study of the distribution of frequencies in band spectrum was first made by H. Deslandres who found that the successive differences in frequency form an arithmetical progression.⁷ Both treatments by Deslandres and by Kayser and Runge were empirical, and were not concerned with the structure of matter, molecules, or atoms.⁸ However, it became clear that the band spectrum is a phenomenon usually arising from molecules and not from isolated atoms.⁹ An early attempt to explain the above fact by the use of an atomic model was made by Nagaoka as follows: When different atoms are combined in a complex molecule, the positive charges can be easily set into vibration perpendicular to the planes of the surrounding rings of those atoms because the charges are very near to each other; the vibration of the positive charge excites the vibration of revolving electrons perpendicularly to the plane, and this produces the band spectrum.¹⁰

E. C. C. Baly, for example, evaluated Nagaoka's model in connection with his analysis of the band spectrum in a book on spectroscopy (1905). Baly pointed out that "about the spectra of compounds very little is known".¹¹ Then he continued as follows:

It has been suggested that each atom of a chemical element consists of a central positively charged mass with the system of negatively charged electrons in motion round about it, very similar to the ring system of planet Saturn with electrical substituted for gravitational attraction. Nagaoka has shown that it is possible to account for both band and line spectra by vibrational disturbances occurring in such a system.¹²

Nagaoka's theory of band spectra is closely connected with his interest in explaining according to his model the magnetic effect on molecules. Although Nagaoka developed his research in band spectra on the basis of his unique theory of the structure of matter, he did not say anything foreign to classical physics. He only stressed the quantitative accord between his formula and that of Deslandres.

It should be added that Nagaoka returned to research in band spectra in 1922 when he published a paper on the band spectra and electron configuration of nitrogen and carbon monoxide molecules, following his experimental research in spectroscopy.¹³

⁷ H. Deslandres, *Comptes Rendu*, **103**, 375-379, 1886.

⁸ H. Kayser, *Handbuch der Spectriscope*, **2**, Herzog, Leipzig, 1902, p. 480; "Spectroscopy," *Encyclopedia Britanica*, 11th ed., **25**, Cambridge, 1911, pp. 619-632.

⁹ H. Kayser, *ibid.* p. 257.

¹⁰ H. Nagaoka, "The Structure of an Atom", *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 245, 1904.

¹¹ E. Baly, *Spectroscopy*, Longmans & Green, London, 1905, p. 425.

¹² E. Baly, *ibid.*, p. 465.

¹³ H. Nagaoka, *Jap. Journ. Phys.* **1**, 49-57, 1922.

3. Dispersion of Light

After the argument with G. A. Schott about the mechanical stability of Saturnian atomic model (1904),¹⁴ on January 20th 1905 Nagaoka published a paper, "Dispersion of Light due to Electron-atoms"¹⁵ (note that Nagaoka used the term "electron-atom" both for his Saturnian atom and for J. J. Thomson's). Here Nagaoka assumed that the positive sphere and the negative electrons circulating about it in the atom are disturbed when a plane electromagnetic wave (light) is incident upon them. Nagaoka wrote the equation of motion for an electron in terms of the displacement of ζ_h , perpendicular to the plane of the circular ring:

$$m \frac{d^2 \zeta_h}{dt^2} = -eZ - \frac{eE}{a_h^3} \zeta_h - \gamma_h \frac{d\zeta_h}{dt}$$

where a denotes the radius of the electron ring, m the mass of electron, Z the electric field of the plane wave, γ_h the mean value of

$$\frac{2}{3} \frac{e^2}{V_0} \frac{\left(\frac{d^2 \zeta}{dt^2}\right)^2}{\left(\frac{d\zeta}{dt}\right)^2}.$$

Here the velocities of light in free ether (vacuum), and in the medium are indicated by V_0 and V , respectively.

Nagaoka, who assumed the central sphere of each atom to be a large rigid particle, wrote the equation of motion for a central particle with mass M_k , and charge E_k :

$$M_k \frac{d^2 \zeta_k}{dt^2} = E_k Z - \frac{\nu e E_k}{a_k^3} \zeta_k - \gamma_k \frac{d\zeta_k}{dt}$$

where a_k^3 denotes the mean value of a^3 , and ν the number of electrons in the atom. It is worth while to point out that Nagaoka regarded the central sphere in his Saturnian atom as a positively charged material particle in continuation of his

¹⁴ Papers on the Nagaoka-Schott argument:

G. Schott, "A Dynamical System illustrating the Spectrum Lines and the Phenomena of Radio-activity", *Nature*, **69**, 437 March 10, 1904.

H. Nagaoka, "Reply to Mr. Schott's Remark on the Motion of Particles in an ideal Atom illustrating the Line and Band Spectra and the Phenomena Radioactivity", *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 140-141, May 17, 1904.

—, "A Dynamical System illustrating the Spectrum Lines", *Nature*, **70**, 124-125, June 9, 1904.

G. Schott, "On the Kinetics of a System of Particles illustrating the Line and Band Spectrum", *Phil. Mag.*, [6], **8**, 384-387, Sept. 1904.

H. Nagaoka, "The Structure of an Atom", *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 240-247, Dec. 18, 1904.

¹⁵ H. Nagaoka, *Proc. Tokyo Phys.-Math.*, [2], **2**, 280-285, 1905.

first paper on his atomic model. This clearly shows that his corpuscular view of electricity plays the most important role in these series of papers.

In the second part of his paper on the dispersion of light Nagaoka assumes a current density W , when light is transmitted through the medium which is filled with ether and his Saturnian atoms. The current density in the z -direction is

$$W = \frac{1}{4\pi} \frac{\partial Z}{\partial t} + \frac{\partial}{\partial t} (-\Sigma e \zeta_h \cos^2(\zeta_h, z) + \Sigma E_k \zeta_k \cos^2(\zeta_k, z))$$

where the summations with regard to ζ_h and ζ_k are to be extended respectively to all the negative electrons and positive particles in unit volume. Then Nagaoka used for the current density the following expression taken from Maxwell's book *A Treatise on Electricity and Magnetism* (1891)¹⁶:

$$W = \frac{K}{4\pi} \frac{\partial Z}{\partial t} + CZ$$

where K denotes the specific inductive capacity, and C the conductivity.

From these equations Nagaoka obtained the formula for the index of refraction n ($n = V_0/V$):

$$n^2 = A + \Sigma \frac{B_h}{\lambda^2 - \lambda_h^2} + \Sigma \frac{B_k}{\lambda^2 - \lambda_k^2}$$

where λ denotes the wave length of incident electromagnetic wave, λ_h that of circular motion of electrons about the central charge, and λ_k that of transverse vibration of the central positive particle. Nagaoka pointed out that this formula is no more than the Ketteler-Helmholtz formula of dispersion.¹⁷

Nagaoka emphasized that the advantage of his method over the Ketteler-Helmholtz formula lay in the discrimination of the periods due to negative and positive charges. However, according to Nagaoka, this discrimination is only effective in the region of the remote infra-red spectrum where one may find the presence of the effect due to positive particles. He wrote as follows:

Usually λ_h will be small, and the term in the formula will be effective when there are numbers of electrons having the same proper period of vibration. We have already seen that λ_k must be quite large; a singularity in the dispersion is accordingly to be expected in the ultra-violet or infra-red region. Although e is very small, the ratio e/m being great compared to E/M , the part played by the negative electrons in causing the dispersion is decidedly greater than that due to positive particles. Especially in the region of visible spectrum, we may suppose that really all the effect is due to negative electrons.¹⁸

¹⁶ J. C. Maxwell, *A Treatise on Electricity and Magnetism*, 3rd ed., 2, Dover, New York, 1891, p. 433.

¹⁷ H. Nagaoka, *Ref. 15*), p. 284.

¹⁸ H. Nagaoka, *ibid.*

Nagaoka's way of thinking was based on Maxwell's electromagnetic theory of light. Therefore he naturally accepted Maxwell's concept of the ether as the medium through which light is transmitted. Nagaoka's theory of propagation of light, however, differed from Maxwell's in the constitution of the medium. Nagaoka assumed that the medium is filled with ether and with his Saturnian atoms. On the contrary, Maxwell had not yet considered the atomic structure of the medium. It was in the late 1910's that Nagaoka referred to Lorentz's theory of electrons to explain the phenomena of dispersion of light.

The above paper by Nagaoka aimed at the justification of his Saturnian atomic model within the framework of classical physics. This defensive attitude led him to prove that one can arrive at a result equivalent to the Ketteler-Helmholtz formula, even by the use of his model. Nagaoka considered classical electrodynamics adequate for describing the behavior of systems of atomic size.

Nagaoka's corpuscular view of electricity required the existence in his atomic model of a positively charged material particle as well as of a negative particle (electron). It is interesting to note that this view does not conflict with his adoption of Maxwell's ether, where both negative and positive particles move freely.

After publishing his paper on dispersion of light, in March, 1906 Nagaoka wrote an analogous paper on geophysics, "Dispersion of Seismic Waves"¹⁹. He dealt with the transmission of seismic waves as if they were light waves. Mountains, lakes and small islands or plains, which lie in the way of seismic waves, give rise according to Nagaoka to a new system of forced and free vibrations, so that the energy of the waves is gradually dissipated and consumed in giving rise to subsidiary motions; therefore the transmission of such waves would resemble that of light waves, which set the molecules and atoms embedded in the ether into forced vibrations and cause dispersion. Nagaoka regarded mountains and other obstacles as behaving like molecules and Saturnian atoms in his analogy. He, however, pointed out the following difference between them:

In light waves, the wave lengths are extremely short, but the embedded molecules partake of the same character; In seismic waves, a wave length may sometimes stretch over hundred kilometres, and consequently a mountain or even a mountain range will occupy a space small compared with a wave length.²⁰

(For Nagaoka's research in geophysics see Mr. Tōsaku Kimura's paper²¹)

In the above paper Nagaoka showed his tendency to treat problems of "macro-cosmic world" on the analogy of the "micro-cosmic world" of atomic problems. His theory of the structure of matter played a most important role in all his

¹⁹ H. Nagaoka, *Proc. Tokyo Phys.-Math. Soc.*, [2], 3, 44-51, 1906.

²⁰ H. Nagaoka, *ibid.*, p. 44.

²¹ T. Kimura, "Nagaoka's Geophysical Studies and their Role in his Physical Researches", in this issue.

researches in physics as the origin of many of his ideas. One may call his research in atomic models and the structure of matter a "source research" while that in geophysics a "sink research". Among Japanese physicists in the Meiji period Nagaoka might be considered as the first physicist to have his own source of ideas even though it was related to his relatively primitive philosophy.

4. Index of Refraction

On February 18th 1905 Nagaoka published a paper, "Relation between the Index of Refraction and Density"²² following his theory of the structure of matter. In his paper on the dispersion of light Nagaoka has shown that the index of refraction n is given by

$$n^2 = A + \sum \frac{B_h}{\lambda^2 - \lambda_h^2} + \sum \frac{B_k}{\lambda^2 - \lambda_k^2}$$

where

$$A = 1 + \frac{4\pi e^2}{V_0^2 m} \sum \lambda_h^2 \cos^2(\gamma_h, z) + 4\pi \sum \frac{E_k^2 \lambda_k^2}{V_0^2 M_k} \cos^2(\zeta_k, z)$$

$$B_h = \frac{e^2 \lambda_h^4}{\pi V_0^2 m} \cos^2(\zeta_h, z)$$

$$B_k = \frac{E_k^2 \lambda_k^4}{\pi V_0^2 M_k} \cos^2(\zeta_k, z) .$$

The summations, as mentioned, are extended over all negative and positive particles, of the substance causing the refraction. Here Nagaoka mentioned that if the density of the substance were to be changed, either by application of pressure, or by temperature variation, or any other agency changing the number of particles, per unit volume the refraction n would be affected to some extent. Considering the fact that the numbers of both negative and positive particles in unit volume are proportional to the density of substance, Nagaoka assumed that the quantities under the sign of summation would all be proportional to the density ρ , if the wave lengths of vibrations of those particles were not changed. In addition, he assumed that $\cos^2(\zeta, z)$ takes all possible values between 0 and 1 for the electrons which play the main part in causing the refraction of light, as mentioned in the previous section. These simple hypothesis, made by Nagaoka, led him to Newton's formula, which is closely satisfied by several substances.

$$\frac{n^2 - 1}{\rho} = \text{const.}$$

Nagaoka continued as follows:

The above formula was deduced on the supposition that the isolated

²² H. Nagaoka, *Proc. Tokyo Phys.-Math.*, [2], 2, 293-295, 1905.

atoms are affected only by the electromagnetic vibrations, while in reality the constituent electrons of each atom will be subject to vibrations due to the mutual interaction of the atoms constituting the molecules, and of the molecules between themselves. Most probably the number of electrons actually contributing to refraction would be only a small number of the whole, while others would be performing irregular vibrations due to the mutual action of atoms and molecules. Evidently the change of density will call forth changes in the internal forces, and on this account introduce changes in the index of refraction.²³

He then discussed in general the effects of temperature and pressure in changing the density in gases, liquids, and solids, but he did not propose any actual experiment to check his theory.

As mentioned above, Nagaoka's ideas on the structure of matter are very vague as to how atoms constitute molecules in substances. Without knowing the actual constitution one can not speak in detail about the mutual action of atoms and molecules. Nagaoka himself felt the necessity of inquiring into these interactions.

5. The interactions of atoms

Following the above line of research, Nagaoka on March 15th 1905 published a paper, "Mutual Action of Electron-atoms."²⁴ In the first part of the paper Nagaoka said as follows:

On several occasion I have discussed the properties of electron atoms, on the supposition that an atom is composed of a central positive charge with a large number of negative electrons moving in circular orbits, all lying in same plane; in other words, the disintegration of an atom gives a single positive particle and a number of electrons. Each atom is supposed to be electrically neutral. Taking any two of these atoms with central charges E and E' , let us consider the action of one atom on another.²⁵

Nagaoka assumed that forces between the first atom (1) with positive charge E and the second atom (2) with positive charge E' are the sum of electrostatic force R_e and magnetic force H . Then he supposed that the electrostatic action on the first atom (1) by the second atom (2) was resolved in four parts:

- I. Repulsion between the two central charges.
- II. Attraction of the positive particle of (1) by the negative electrons of (2).
- III. Attraction of negative electrons of (1) by the positive particle of (2).
- IV. Repulsion of the negative electrons of (1) by those of (2).

In the first place Nagaoka mentioned that the repulsion between the central

²³ H. Nagaoka, *ibid.*, p. 294.

²⁴ H. Nagaoka, *Proc. Tokyo Phys.-Math.*, [2], 2, 316-320, 1905.

²⁵ H. Nagaoka, *ibid.*, p. 316.

positive particles with the charges E and E' at distance r was given by

$$\frac{EE'}{r^2}.$$

In finding the action of one electron ring on another positive particle or on another ring, Nagaoka supposed that the electrical ring with a mean radius a was uniformly distributed with a linear density $-E/2\pi a = \rho$. Thus the electrical potential of the circular ring of radius a and the density ρ at a point on the axis can be found. Further, the potential of the ring at points not on the axis but at distance r from the center in a line at an angle θ with the axis is given by

$$V = 2\pi\rho\left\{\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}P_2(\theta) + \frac{1.3}{2.4}\frac{a^5}{r^5}P_4(\theta) - \dots\right\}.$$

Nagaoka obtained the attraction of the ring by another positive particle E' :

$$-E'\frac{\partial V}{\partial r}.$$

He also obtained the attraction of the positive particle E by the another negative electron ring of atom (2) as $-E(\partial V'/\partial r)$. Similarly, Nagaoka deduced the repulsion between the two electron rings in the direction of the radius vector r . Adding all the actions, he found the mutual electrostatic action R_e . Nagaoka wrote about the condition for distance r , namely, "the atoms are sufficiently distant from each other."²⁶ This is related to Nagaoka's corpuscular view of electricity and the condition is essential to avoid inter-penetration between electrical charges (central particles) in his Saturnian atoms. He showed that the electrostatic force is nearly inversely proportional to the fourth power of the distance between the two atoms. He indicated it as $R_e \div \mu/r^4$. Then he added that "they may be attractive or repulsive according to the configuration and size of atoms."²⁷ He, however, did not discuss the actual configuration and size.

In the second place, Nagaoka calculated the magnetic action between the two electron rings of atoms (1) and (2) by the use of J. J. Thomson's calculation for the moment of a small magnet: According to Thomson the magnetic moment of an electron moving in a circle of radius a with angular velocity ω is equivalent to that of the small magnet whose magnetic moment is given by $ea^2\omega$ in the direction normal to the plane of orbit. Consequently, Nagaoka stated "if we suppose n electrons to be moving in a mean circle with mean velocity Ω , they will form an elementary magnet

$$nea^2\Omega = Ea^2\Omega.$$

The atoms will thus be equivalent to a magnet of mean moment $Ea^2\Omega$ in

²⁶ H. Nagaoka, *ibid.*, p. 318.

²⁷ H. Nagaoka, *ibid.*

electrostatic units."²⁸

Nagaoka supposed that the magnetic action between the two atoms was similar to that of two elementary magnets. Thus he deduced that the magnetic action H was approximately inversely proportional to the fourth power of the distance between the atoms. Here he again added that it might be attractive or repulsive according to the position of atoms. However, he did not discuss the actual position of the atoms.

In the last part of his paper Nagaoka concluded:

Thus, on the whole both the electrostatic and the magnetic actions give rise to forces proportional to the inverse fourth power of the distance between the atoms, when they are at a distance sufficiently great compared with their dimensions.²⁹

Further, Nagaoka said that "the molecules being formed of a number of atoms, the molecular action will take place according to the inverse fourth power of the distance, and will be jointly proportional to the respective masses."³⁰ He made a note that W. Sutherland in 1893 arrived at a similar conclusion on the molecular action from an entirely different standpoint.

It is interesting to discuss why Nagaoka had to treat the problem of the mutual action of atoms under the condition that the distance between the atoms should be sufficiently great compared to their dimension. The reason why he had to regard the force as action at a distance may well be explained by Nagaoka's rejection of the inter-penetrability of electrical charges. As mentioned above, having considered the central positive charge as a rigid particle, he arranged the electrons outside this particle in his model.

According to an interesting paper by Takehiko Takabayashi on the Kelvin-Thomson atomic model, this model has an advantage in giving an adequate explanation for chemical attraction although it is only qualitative. The advantage is closely connected with the acceptance of inter-penetrability between electric charges in the atoms. Takabayashi in 1951 remarked:

The spherical charge in the Kelvin-Thomson atomic model may be electricity itself, which is not associated with any material substance, or if one assumes any electrically charged substance then the substance must lack impenetrability, which may be a fundamental property of ordinary material substances.³¹

As regards Nagaoka's analogical way of thinking, it is worthwhile to refer to his papers on the coil, "Note on the Potential and the Lines of Force of a

²⁸ H. Nagaoka, *ibid.*, p. 319.

²⁹ H. Nagaoka, *ibid.*

³⁰ H. Nagaoka, *ibid.*

³¹ T. Takabayashi, "Kadenun-genshi I" (Atoms with Charge Cloud), *Shizen (nature)*, 6, No. 7, 77, 1951.

Circular Current”³² and “The Coefficient of Mutual Induction of Two Coaxial Circular Coils,”³³ published in 1903 and 1904 respectively. Here Nagaoka calculated the magnetic potential of a circular electrical current and the coefficient of mutual induction of two coaxial circular coils, starting from Maxwell's book, *A Treatise of Electricity and Magnetism*, Vol. 2, Chapter XIV of circular currents. Using Legendre's table Maxwell had calculated the coefficient of mutual induction of two coils (or the mutual potential energy denoted by M). On the other hand, Nagaoka calculated M by the use of a rapidly converging q -series, which is convenient for practical purpose. (Note that the force X acting in the direction of the displacement dx is given by dM/dx , if any displacement alters the value of M .) These papers on coil seem to have analogies with his paper on the mutual action of atoms.

Nagaoka's research on the interactions of atoms remained qualitative, and was not developed into a quantitative approach to the actual structure of atoms and molecules. On the other hand his research on coils was continued for more practical purposes. Eventually, two tables, namely “Tables of Theta-Functions, Elliptic Integrals K and E , and associated Coefficients in the Numerical Calculation of Elliptic Functions,”³⁴ and “Tables of the Self-Inductance of Circular Coils and of the Mutual Inductance of Coaxial Circular Currents”³⁵ were published in 1922 and 1926 respectively. These tables have been used among electrical engineers.

6. Virial of Molecular Forces

As the continuation of his paper on the mutual action of atoms Nagaoka published a paper, “Virial of Molecular Forces due to Electron Atoms, the Characteristic Equation and the Joule-Kelvin Effect”³⁶ on April 25th 1905. Starting from R. Clausius virial theorem, Nagaoka showed in the first part of the paper the possibility of obtaining Van der Waals' equation even if one assumes that molecules consist of his Saturnian atoms. Nagaoka mentioned the virial equation due to Clausius:

$$nm\frac{\overline{c^2}}{2} + \frac{1}{2} \sum (\overline{X_h x_h + Y_h y_h + Z_h z_h}) = 0$$

where $\overline{c^2}$ denotes the mean square velocity, m the mass of a molecule, n the number of molecules of the system. The expression—in the second term indicates the time average. X_h , Y_h , and Z_h denote the components of the total force acting on the h th molecule, and x_h , y_h , z_h the coordinates. Twice the second term,

³² H. Nagaoka, *J. Coll. Sci. Tokyo*, **16**, Art. 15, 1-16, 1903.

³³ H. Nagaoka, *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 233-239, 1904.

³⁴ H. Nagaoka (with S. Sakurai), *Sci. Pap. I. P. C. R.*, Table No. 1, 1-67, 1922.

³⁵ H. Nagaoka (with S. Sakurai), *Proc. Imp. Acad.*, **3**, No. 1, 19-22, 1926.

³⁶ H. Nagaoka, *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 335-340, 1905.

which is called the virial of the forces, is equal to the summation of the virial of the external forces W_a and of that of the internal forces W_i . Thus the above equation becomes

$$nm\overline{c^2} + W_a + W_i = 0$$

Nagaoka writes that W_a is equal to W_1 , and W_i the summation of W_2 and W_3 under the assumption that the forces acting on molecules of the system are divided into the following three parts:

1. The pressure acting on the molecules.
2. Forces acting during impact.
3. Forces acting according to inverse fourth power of the distance between the molecules.

Further he continued:

- (a) The virial due to pressure p is well known to be

$$W_1 = -3pV$$

where V is the volume of gas.

- (b) The virial due to the impact of spherical molecules of diameter σ was shown to be

$$W_2 = nm\overline{c^2} \frac{b}{v} \left(1 + \frac{5b}{8v} \right)$$

where v is the specific volume, namely $v = V/nm = V/\rho$,

$$b = \frac{2\pi\sigma^3}{3m}$$

and ρ the density.

The virial due to internal molecular force was calculated according to Nagaoka's research, mentioned above: He obtained the result that the attraction between two molecules is $-\mu/r^4$. It may be useful to quote a passage from his paper on the virial of molecular forces to show his approach:

The number of molecules in a volume element $d\tau$ is $(\rho/m)d\tau$ and in another element $d\tau'$ is similarly $(\rho/m)d\tau'$. The attraction between any two molecules being $-\mu/r^4$, the virial of molecular forces in the two elements $d\tau$ and $d\tau'$ is

$$-\mu\rho^2 \frac{d\tau d\tau'}{m^2 r^3}.$$

The virial of internal forces on molecules in $d\tau'$ due to remaining molecules is³⁷

$$\frac{\mu\rho^2 d\tau'}{m^2} \int \frac{d\tau}{r^3}.$$

³⁷ H. Nagaoka, *ibid.*, p. 336.

Then Nagaoka calculated the integral of $d\tau/r^3$ as follows:

$$\int_{(\text{Sphere})} \frac{d\tau}{r^3} = \iiint \frac{\sin \theta d\theta d\varphi dr}{r} = 2\pi \log \frac{R}{r}$$

were R indicates the maximum distance from $d\tau'$, which is of the order of 0.1 cm, and r the minimum distance of the order of 10^{-8} cm, namely the atomic dimension. Thus the virial of the internal molecular forces becomes $W_3 = 3\rho^2 aV$, where $a = (2\pi/3)\mu \log(R/r)$. Putting these virials W_1 , W_2 , and W_3 in the above equation, Nagaoka showed that the virial equation takes the form

$$nmc^2 - 3pV + 3\rho c^2 \frac{b}{v} \left(1 + 5\frac{b}{v}\right) + 3\rho^2 aV = 0.$$

Identifying the mean square velocity $\overline{c^2}$ with $3B\theta$, which is reduced from the kinetic theory of gases, and substituting $1/\rho$ for $V/nm = v$, Nagaoka obtained the equation:

$$p + \frac{a}{v^2} = \frac{B\theta}{v} \left(1 + \frac{b}{v} + 5\frac{b^2}{v^2}\right)$$

Nagaoka emphasized that the above equation can be written in the form of Van der Waals' characteristic equation if one neglects quantities of order b^2/v^2 :

$$\left(p + \frac{a}{v^2}\right)(v - b) = B\theta$$

The concept of the virial was first introduced into the theory of gases by Clausius, as Nagaoka mentioned in his paper. However, Nagaoka's calculation itself seems to be based mainly on Boltzmann's work, although Nagaoka does not make any reference to his teacher in Germany. Boltzmann dealt in 1898 with the problem of the derivation of Van der Waals' equation by means of the virial concept in his fifth chapter of *Vorlesungen über Gastheorie* II. Boltzmann had already derived Van der Waals' equation considering virials of the external pressure, of the impact of molecules, and of Van der Waals cohesion force, each of which nearly corresponds to W_1 , W_2 , W_3 respectively in Nagaoka's notations. Nagaoka's way of calculating W_1 , and W_2 is the same as Boltzmann's. The former's way for W_3 , however, differs from the latter's in the form of the molecular attraction. Nagaoka assumes it as $-\mu/r^4$ while Boltzmann simply writes $-F(r)$.³⁸ This is based on their different assumptions on the structure of atom. Evidently, Nagaoka utilized his atomic model with the central positive particle and a number of electrons, while Boltzmann had not considered any internal structure of atom.

³⁸ L. Boltzmann, *Vorlesungen über Gastheorie*, 2, Leipzig, 1896, & 1898; trans. by S. B. Brush, *Lectures on Gas Theory*, Univ. of California Press, California, 1964, p. 353.

Boltzmann obtained

$$W_3 = 3\rho^2 aV$$

where

$$3a = \frac{1}{m^2} \int d\omega r F(r) .$$

He explained that "since this (the right term) depends only on the nature of function F , it must be a constant peculiar to the substance, and we denote it by $3a$."³⁹

It would be quite easy to notice that the form of W_3 is the same in both Nagaoka's and Boltzmann's works. In Nagaoka's paper the letter a is not a constant but a function of $\log(R/r)$, since it stands for $(2\pi/3)\mu \log(R/r)$. Nagaoka, however, regards a as almost a constant in the first part of his paper because the logarithm of such a large quantity as R/r causes only a small increase or decrease of R or r . Therefore Nagaoka's derivation of Van der Waals' equation is almost the same as Boltzmann's. Nagaoka showed that the equation would be produced without any difficulty, even if one assumes his Saturnian atoms. Nagaoka mentions that "so far nothing particularly worth noticing has been obtained by the consideration of electron-atoms, except a clearer understanding of the role of the constant a in the characteristic equation."⁴⁰

Nagaoka had assumed a and b as constants, but later in the paper he said that "the internal commotion which necessarily accompanies the molecular arrangement of atoms will affect these quantities, so that they are in the strict sense not constants."⁴¹ Therefore he discussed $\partial b/\partial\theta$, $\partial a/\partial\theta$, and $\partial\theta/\partial p$ in the second part of his paper on the virial of molecular forces, starting from Van der Waals' equation. Finally, a somewhat more complex expression for the Joule-Kelvin effect (Joule-Thomson effect) is given.

7. Conclusions

From our analysis of a series of theoretical papers in which Nagaoka deals with the problems of band spectra, the dispersion of light, the index of refraction, the interaction of atoms, and of the virial of molecular forces, the following remarks could be made on the general characteristics of Nagaoka's theory of structure of matter⁴²:

(1) It is assumed in Nagaoka's theory of the structure of matter that molecules consist of a number of his Saturnian atoms.

(2) His atom consists of a number of electrons of equal mass, arranged

³⁹ L. Boltzmann, *ibid.*

⁴⁰ H. Nagaoka, *Ref. 36*) p. 337.

⁴¹ H. Nagaoka, *ibid.*, p. 338.

⁴² E. Yagi, "H. Nagaoka's Theory of Structure of Matter (1904-05)," *Actes XIII^e Congrès International D'Histoire Des Sciences, Moscow 1971* (in press).

uniformly in a circular ring, and a positively charged sphere of large mass, located at the center of the ring. Rejecting inter-penetrability between two kinds of electricity, he arranges negative electrons outside the central positive charge. This so-called "corpuscular view of electricity" played a most important role in his theory of structure of matter.

(3) On the assumption that both negative and positive charges are material particles, forces among those particles, namely the interaction of atoms, are regarded as essentially action at a distance. (Note that in his Saturnian atom Nagaoka used J. Lamor's work to solve the difficulty of the exhaustion of energy of radiation from revolving electrons. See *Ref. 1*).

(4) Thus the behavior of each particle is treated by solving its equation of motion. The theory of vibration was frequently used in Nagaoka's papers.

(5) On the other hand, little quantitative consideration is paid to the actual constitution of molecules. Nagaoka deals with molecules in general and not with specific molecules, such as hydrogen. His interest in the chemical nature of atoms and molecules was much weaker than J. J. Thomson's.

(6) Apart from his unique view of electricity, Nagaoka had a tendency to treat problems of structure of matter, as if he had solved problems of mathematical physics by the use of standard European textbooks.

(7) Nagaoka's way of thinking was closely connected with his studies in Germany from 1893 to 1896. He was particularly influenced by L. Boltzmann,* who tried to explain physical phenomena by the use of a mechanical model.

(8) Nagaoka also had a tendency to treat problems of "the macro-cosmic world", namely geophysical and cosmological problems, by the analogy of those of the "micro-cosmic world" atomic problems. Therefore, his theory of structure of matter played the most important role in his whole research in physics as the origin of his ideas.

(9) Nagaoka had a strong motivation to justify his atomic model within the framework of classical physics. This attitude induced him to prove that his theory of the structure of matter does not conflict with any previous theory of physics. This means that one can arrive at a result equivalent to previous formulae such as Deslandres' formula for band spectra, Helmholtz-Kettler's dispersion formula, Newton's formula for the index of refraction and density, Sutherland's formula for molecular force, and Van der Waals's equation of state, even by the use of Nagaoka's atomic model.

(10) On the other hand, the above attitude prevented Nagaoka from finding new phenomena which may be suggested by the introduction of his atomic model into classical physics.

(11) Nagaoka's corpuscular view was mainly limited to electricity and

* Through Boltzmann, Nagaoka was also influenced by Maxwell.

mass, and was not extended to energy. Nagaoka hesitated in the 1900's to accept Planck's h and Einstein's light quantum even though Planck had been one of his teachers in Germany.

Nagaoka started to do experimental research in spectroscopy in 1908. The reason why Nagaoka stopped carrying out his theory on the basis of his atomic model after 1905, is as follows: Nagaoka's atomic model contains a great number of electrons, 10^4 electrons in general. Only a few number of them are arranged at the outer-most ring concerned. Thus the mechanical stability of his system is indispensably connected with the number. However, J. J. Thomson proved in 1906 that the number of electrons in an atom of an elementary substance is the same order as the atomic weight of the substance, and that it is not as great as previously proposed. We found that Nagaoka himself referred to it; he wrote in 1909 in a Japanese paper that one can not assume the order of 10^4 electrons in mercury atom any longer because the recent progress of theory of electrons has proved that the number of electrons in an atom is the same order of the atomic weight.⁴³ This suggests that after the foundation for mechanical stability of his atomic model had been destroyed, Nagaoka gave up the development of his theory on its basis.

The reason why Nagaoka started spectroscopic experiments can be understood by his having the following recognition; he wrote in the same Japanese paper that one can obtain a knowledge of the real arrangement of electrons in an actual atom only through these experimental investigations.

After the renunciation about 1907 of his Saturnian atomic model, Nagaoka soon adopted the hypotheses of inter atomic magnetic fields from W. Ritz's paper, "Magnetische Atomfelder und Serienspektren,"⁴⁴ published in 1908. This hypothesis played a most important role in Nagaoka's spectroscopic researches between 1908 and 1923. Using the analogy of Zeeman effect by the external magnetic field, Nagaoka tried to explain the regular distribution of the satellites of the mercury lines. He assumed that a principal line is divided into these satellites by the internal magnetic field in an atom.

I shall report on the development of Nagaoka's research in spectroscopy at some future date.

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⁴³ H. Nagaoka, "Genshi no Mokei IV" (on atomic models), *Tōyō Gakugei Zasshi* (journal of arts and science in the Orient), 26, pp. 310-314, 1909.

⁴⁴ M. Ritz, *Ann. der Phys.*, [4], 25, 660-690, 1908.

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Appendix I

A List of Nagaoka's series of papers on his Saturnian Atomic Model* (1904–05)

1. "Motion of Particles in an ideal Atoms illustrating the Line and Band Spectra and the Phenomena of Radioactivity", *Proc. Tokyo Phys.-Math. Soc.*, [2], **2**, 92–107, Feb., 1904 (Read Dec. 5, 1903).
2. "On a Dynamical System illustrating the Spectrum lines and the Phenomena of Radioactivity", *Nature*, **69**, 392–393, Feb. 25, 1904 (Written Jan. 18)
3. "Extension of Deslandres' Formula for Band Spectrum", *Proc. Tokyo Phys.-Math. Soc.*, [2] **2**, 129–131, April 20, 1904 (Read March 19).
4. "Kinetics of system of Particles illustrating the Line and Band Spectrum and the Phenomena of Radioactivity", *Phil. Mag.*, [6], **7**, 445–455, May, 1904.
5. "Über ein die Linen- und Bandenspektren, sowie die Erscheinungen der Radioaktivität veranschaulichendes dynamisches System", *Phys. Zeit.*, **5**, 517–521, 1904.
6. "Dispersion of Light due to Electron-Atoms", *Proc. Tokyo Phys.-Math.*, [2], **2**, 280–285, Jan. 20, 1905 (Read Dec. 17, 1904).
7. "Relation between the Index of Refraction and Density", *ibid.*, 293–295, Feb. 18, 1905 (Read Jan. 21).
8. "Mutual Action of Electron Atoms", *ibid.*, 316–320 March 15, 1905 (Read Feb. 19).
9. "Virials of Molecular Forces due to Electron Atoms, the Characteristic Equation and the Joule-Kelvin Effect", *ibid.*, 335–340, April 25, 1905 (Read March 18).

* For the Nagaoka-Schott argument see *Ref. 14*).