Newton's *Quantitas Materiae*

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It was more than ten years ago that one of the present authors, Masao Watanabe, published his view concerning Newton's concept of mass revealed in the *Principia*.[1] Commenting on this paper, a different opinion was then presented by Kiyonobu Itakura.[2] Watanabe replied to Itakura later in another paper confirming and strengthening his original point.[3] The present paper will first review this academic dispute, will then refer to John Herivel's more recent work, and will end with the presentation of some of the results of the recent studies made by the present authors.

Newton, in the first of the eight Definitions at the beginning of the *Principia*, defined "the quantity of matter" (*quantitas materiae*) as follows:

> The quantity of matter is the measure of the same, arising from its density and bulk conjointly.[4]

As cited by Florian Cajori, Ernst Mach criticized this definition as being circular, since density can only be defined as the mass of unit volume; but Cajori also noted that Henry Crew held that "it is both natural and logically permissible to define mass in terms of density" at the time of Newton when "the density of water was taken arbitrarily to be unity"; whereas Edmund Hoppe, according to Cajori, assumed that Newton's atoms were of the same size and consequently the densities of bodies were proportional to the numbers of such atoms in equal volumes.[5] Cajori himself was critical of Hoppe's interpretation because it did not accord with Newton's corpuscular idea as described in his *Opticks*.[6] E. A. Burtt, W. C. Dam-

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1 Masao Watanabe, "Newton's Concept of Mass as revealed in his *Principia*," *Kagakusi Kenkyu* (*Journal of History of Science, Japan*), No. 54, 1960, pp. 1–4.

2 Kiyonobu Itakura, "Newton's Definition of Mass in *Principia* and Galilei's Atomism in *De Motu*," *Kagakusi Kenkyu*, No. 59, 1961, pp. 29–31.

3 Masao Watanabe, "Newton's Concept of Mass—A Reply to Dr. Itakura," *Kagakusi Kenkyu*, No. 84, 1967, pp. 191–194.

4 Isaac Newton, *Mathematical Principles of Natural Philosophy*, Motte-Cajori translation, Berkeley, 1947, p. 1.

5 Ernst Mach, *Die Mechanik in ihrer Entwicklung* (ed. 8), Leipzig, 1921, p. 188; Henry Crew, *The Rise of Modern Physics*, Baltimore, 1928, p. 124; Edmund Hoppe, *Archiv für Geschichte der Mathematik, der Naturwissenschaften und der Technik*, n.s., Vol. 11, 1929, pp. 354–361 (according to Cajori's Appendix, Newton's *Principles, op. cit.*, pp. 638–639).

6 Isaac Newton, *Opticks*, 3rd ed., 1721, pp. 375–376 (Cajori, *Ibid.*).

pier, and Max Jammer, on the other hand, suggested that Newton's definition of mass as the product of density and bulk was under the influence of Boyle's law, in which the quantity of the gas under varying pressure was determined by the product of its voluem and density.[7]

The above mentioned might well explain why Newton's definition of mass took that particular form, but did not go deep enough into Newton's concept of mass itself, according to Watanabe. He claimed, therefore, that Newton's concept must be clarified from Newton's own writings and he suspected that the clue of the matter consisted in the latter part of Newton's comment to Definition I, which reads:

> And the same [quantity of matter] is known by the weight of each body, for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shown hereafter.[8]

These experiments on pendulums are again mentioned in Corollary VII to Proposition XXIV, Book II, that is:

> And by experiments made with the greatest accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.[9]

These experiments are more fully described in Proposition IV, Book III:

> It has been now for a long time, observed by others, that all sorts of heavy bodies (allowance being made for the inequality of retardation which they suffer from a small power of resistance in the air) descend to the earth *from equal heights* in equal times; and that equality of times we may distinguish to a great accuracy, by the help of pendulums. I tried experiments with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I provided two wooden boxes, round and equal: I filled the one with wood, and suspended an equal weight of gold (as exactly as I could) in the centre of oscillation of the other. The boxes, hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally receiving the resistance of the air. And, placing the one by the other, I observed them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by Cor. I and VI, Prop. XXIV, Book II[10]) was to the quantity of matter in the wood as the action of the motive force (or *vis motrix*) upon all the gold to the action of the same upon all the wood; that is, as the weight of the one to the weight of the other: and the like happened in the other bodies. By these experiments, in bodies of the

[7] E. A. Burtt, *The Metaphysical Foundations of Modern Physical Science*, Garden City, N.Y., 1954, p. 241; W. C. Dampier, *A History of Science and its Relations with Philosophy and Religion*, Cambridge, England, 1949, p. 155; Max Jammer, *Concepts of Force*, Cambridge, Massachusetts, 1957, p. 118.

[8] *Principles, op. cit.*, p. 1.

[9] *Ibid.*, p. 304.

[10] Concerning the relationship of the mass, weight, period of oscillation, and the length of a single pendulum. *Ibid.*, pp. 303–304.

same weight, I could manifestly have discovered a difference of matter less than the thousandth part of the whole, had any such been.[11]

Newton analyzed the old problem of falling bodies from an entirely new point of view, namely from the aspect of his new dynamics. He saw that since all falling bodies are in fact equally accelerated so far as the resistance of the air may be neglected, the force which causes this equal acceleraton in each body, which is nothing but the weight of the body, must be proportional to the quantity of matter or inertial mass in the same body, as may be inferred from Newton's Law II.[12] To see whether a precise proportionality of this kind between weight and mass exists in various sorts of material, he had to appeal to experiments, since there exists no other means than to empirically establish this proportionality. Therefore, Newton performed experiments on pendulums, which may be considered as a far more accurately designed type of experiment on falling bodies. He tried these experiments with various sorts of materials conceivable, including, interestingly enough, wheat, which has weight and falls down like other materials but possibly goes upward also when it sprouts out.

Thus, behind his seemingly circular or equivocal presentation of the definition of mass, there existed a very clear and novel concept of mass, mass as inertial mass, explicitly distinguished from weight. In this connection, moreover, his blurred definition of mass itself as the product of density and bulk of a body may also become intelligible, if we read it together with the following passage of Newton's in Corollary IV to Proposition VI, Book III:

By bodies of the same density, I mean those whose inertias are in the proportion of their bulks.[13]

Once such a proportionality was empirically established and was proved to conform with the entire system of the theories, then there may have been little practical need to make deliberate distinction between inertial mass and weight or gravitational mass. But, without first conceiving the quantity of matter afresh as the measure of inertia inherent to a body but distinct from its weight, neither the whole establishment of Newtonian dynamics nor the formation of the new concept of weight as universal gravitation would have been possible. The concept of mass here played a really important role.

This very clear and original concept of mass of Newton's own was, however, set forth in a very ambiguous presentation. The reason may have probably been related, at least partly, to the following fact. Although physics is an inductive science by nature and the definitions of physical quantities can be given only operationally, Newton presented his new system of physics in a form of deductive science, such as Euclid's geometry, and started the whole book with "Definitions" and "Axioms, or Laws of Motion." It was therefore necessary for him to supplement these

11 *Ibid.*, p. 411.

12 Newton's own argument to the same effect as to pendulums is seen in his Proposition XXIV and immediately following Corollaries, *Ibid.*, pp. 303–304.

13 *Ibid.*, p. 414.

Definitions and Laws with closely following comments and Scholia and to give further explantions in the latter part of his work to show how these defined quantities and axiomatically presented laws are in reality connected with physical phenomena. Thus, notwithstanding the clearness of his concept of mass, Newton might well have written down his definition of mass under the influence of contemporary ideas of atoms, corpuscles, density, or of Boyle's law.

The above is the substance of Watanabe's paper published early in 1960.[14] To this paper, Itakura wrote his own comments,[15] in which he stressed the importance of the influence of atomism on Newton's Definition I and in this connection he held that Newton's concept of mass was of more gravitational character originating in atomism, while admitting that Newton had reached to a very clear idea of inertial mass.

Itakura appealed to Galileo's *De Motu* as evidence. In this work, Galileo gave a distinction between apparent weight and "natural and intrinsic weight" of a body and thought of the latter in connection with the density and the bulk of matter, where density was related to the number of particles in a given volume of the matter.[16] This must have been the common place of atomists of the time including Newton himself, Itakura argued, and he concluded that Newton's mass defined in Definition I must have been mass of atomism and mass to be determined by weight rather than inertia.

To this comment of Itakura's, Watanabe replied somewhat later.[17] He claimed that, while he was concerned mainly about how Newton thought of mass, Itakura was more concerned about how Newton wrote down its definition. So far as Newton's concept was concerned, Newton thought of mass unequivocally as inertial, Watanabe reaffirmed, by citing relevant passages from the text of the *Principia* and elsewhere, which included the following two passages from different sources:

> *Definition 15.* Bodies are denser when their inertia is more intense, and rarer when it is more remiss.[18]
>
> Nor that I affirm gavity to be essential to bodies: by their *vis insita* I mean nothing but their inertia. This is immutable. Their gravity is diminished as they recede from the earth.[19]

In supplementing the above, Watanabe indicated that if, as Itakura pointed out, Galileo's theory of matter in his *De Motu* be appropriate to interpret Newton's definition of mass, then Galileo's descriptions of his experiments on pendulums

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[14] Watanabe (1960), *op. cit.*

[15] Itakura, *op. cit.*

[16] Galileo Galilei, *On Motion and On Mechanics*, tr. by I. E. Drabkin & Stillman Drake, Madison, 1960, pp. 15 & 88 (*Le Opere di Galileo Galilei*, Vol. 1, pp. 252–253 & 318).

[17] Watanabe (1967), *op. cit.*

[18] *De Gravitatione et Aequipondio Fluidorum*, A. R. Hall & M. B. Hall, *Unpublished Scientific Papers of Isaac Newton*, Cambridge, England, 1962, p. 150.

[19] Newton's comment to Rule III, *Principles, op. cit.*, p. 400.

in his *Dialogo* and *Discorsi*[20] must have more pertinence. These experiments can well be the immediate precedent for Newton's, since they are the means Galileo invented to investigate the free fall phenomena more accurately and since the concept of mass there exhibited seems to be anticipating Newton's.

Watanabe now furthered his original view by incorporating Max Jammer's new interpretation as presented in his just published book, *Concepts of Mass*,[21] and he proposed in the same paper that Newton's concept of mass as inertial possibly went down to the level of atoms of which matter was composed. Thus, Newton may have thought of density as the sum of the inertial mass of the constituent atoms or particles in a unit volume of the matter in question, and consequently he defined mass as the product of such density and bulk, which interpretation makes his Definition I more intelligible.

Thus far, the present authors have reviewed two papers of Watanabe's and one related paper of Itakura's. The consequences are in very good accordance with the results of more recent and documented work of John Herivel who has made careful studies of those Newtonian manuscripts which reveal the making of the *Principia*.[22] Herivel points out:

> A brief discussion of the use of pendulum experiments to prove this proportionality occurs at Def. 11 of MS. Xa [1684–85],[23] followed by a much fuller discussion at Def. 7 of MS. Xb [1684–85][24] where the use of *pondus* as an alternative to quantity of matter or mass, even in the absence of any gravitational effects, is brought out much more clearly than in the *Principia*.[25]

Herivel expresses his view concerning Newton's concept and definition of mass as follows:

> However, this flaw in the definition of quantity of matter in no way interfered with the development of Newton's thought in the *Principia*, for when he had to compare the quantities of matter of two bodies he did so by making an appeal to his second law of motion. It is this law, therefore, rather than Def. I, which must be regarded as giving the real, operational definition of mass in Newton's dynamics.
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> As to the original development of the concept, we can be confident that the obvious fact of inertia, or *Vis Insita*, would inevitably have led Newton to attach a measure to it. For example, collision experiments with bodies of

[20] Galileo Galilei, *Diologue Concerning the Two Chief World Systems*, tr. by Stillman Drake, Berkeley, 1962, pp. 151–152; *Dialogues Concerning Two New Sciences*, tr. by Henry Crew & Fonso de Salvio, New York, 1914, pp. 84–87.

[21] Max Jammer, *Concepts of Mass in Classical and Modern Physics*, New York, 1961, pp. 64–74.

[22] John Herivel, *The Background to Newton's Principia*, Oxford, 1965, 337 pp.

[23] "Definition 11" is wholly quoted later in the text of the present paper, p. 33.

[24] This discussion in *De Motu Corporum* is wholly quoted later in the text of the present paper, p. 33.

[25] Herivel, *op. cit.*, p. 25.

equal size, but of different material, revealed that the inertias of such bodies were not entirely determined by their sizes. Two bodies of equal size could have quite different capacities to sustain their states of rest or motion. That he then attempted to give a definition of mass independently of the second law was understandable, though perhaps a trifle unfortunate.[26]

To all that mentioned in the above, the present authors want to add a further and a new conjecture of their own which, they believe, will make Newton's Definition I even more understandable.

This definition of mass in reference to density and bulk has hitherto been explained as mainly owing to the atomistic, or rather corpuscular idea of matter of the time. It may well be so. But, is it not more probable and more immediate to Newton, that he, when making the definition of the quantity of matter, thought of it as the product conjointly of density as intensity and bulk as extension? Such an idea of perceiving a certain quantity as the product of intensity and extension was developed in the tradition of the Merton School,[27] and Newton must have taken it over. In fact, he wrote in his *De Gravitatione et Aequipondio Fluidorum* (MS. Add. 4003) [1664–69]:

> Moreover, the quantity of these powers, namely motion, force, *conatus*, impetus, inertia, pressure and gravity may be reckoned in a double way: that is, according to either intension or extension.
>
> *Definition 11.* The intension of any of the above-mentioned powers is the degree of its quality.
>
> *Definition 12.* Its extension is the amount of space or time in which it operates.
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> *Definition 13.* Its absolute quantity is the product of its intension and its extension. So, if the quantity of intension is 2, and the quantity of extension 3, multiply the two together and you will have the absolute quantity 6.
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> Moreover, it will help to illustrate these definitions from individual powers.... And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body. So force, *conatus*, impetus or inertia are more intense as they are greater in the same or an equivalent body: they have more extension when the body is larger, and their absolute quantity arises from both.... And the absolute quantity [of pressure] results from the intension of the pressure and the area of the surface pressed..., and absolutely speaking the quantity of gravity is the product of the specific gravity and bulk of the gravitating body.[28]

Here, the absolute quantity of motion, impetus, inertia, pressure, gravity, and so on, is thought to be determined by the product of their intensity and extension.

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[26] *Ibid.*, pp. 25–26.

[27] Marshall Clagett, *The Science of Mechanics in the Middle Ages*, Madison, 1961, pp. 199–219.

[28] Hall, *op. cit.*, pp. 114–115 & p. 149, (Herivel, *op. cit.*, p. 225 & pp. 231–232).

In a later manuscript, *De Motu Corporum in medijs regulariter cedentibus* (MS. Add. 3965) [1684–85], Newton wrote:

> Definition 11 The quantity of motion is that which arises from the velocity and quantity of a body conjointly. Moreover the quantity of a body is to be estimated from the bulk of the corporeal matter which is usually proportional to its gravity. The oscillations of two equal pendulums with bodies of equal weight are counted, and the bulk of matter in both will be inversely as the number of oscillations made in the same time.[29]

In *De Motu Corporum* (MS. Add. 3965) [1684–85] he said:

> 7. By the heaviness of a body I understand the quantity or bulk of matter moved apart from considerations of gravity as often as it is not a matter of gravitating bodies. To be sure the heaviness of gravitating bodies is proportional to their quantity of matter by which it can by analogy be represented or designated. The analogy can actually be inferred by equal pendulums as follows. The oscillations of the same weight are counted and the bulk of matter in each case will be inversely as the number of oscillations made in the same time. But careful experiments made on gold, silver, lead, glass, sand, common salt, water, wood, and wheat led always to the same number of oscillations. On account of this analogy and lacking a more convenient word I represent and designate quantity of matter by heaviness, even in bodies in which there is no question of gravity.[30]

It should be noticed here that Newton's definition of the quantity of motion in the above quoted Definition 11 is a modification of the one described in the immediately preceding quotation and is already very much similar to Definition II, *Principia*, which reads:

> The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.[31]

Therefore, on the basis of the following facts:

(1) that the quantity of motion was originally conceived as the product of the velocity as intensity and the magnitude of the body as extension,

(2) that each of such quantities as impetus, inertia, etc. was also thought of to be determined by the product of its intensity and extension,

(3) that the wording of Definition II, *Principia*, quoted in the above is exactly the same as that of its preceding Definition I which defines mass as being inertial and which reads:

> The quantity of matter is the measure of the same, arising from its density and bulk conjointly.[32]

(4) and that the very important pendulum experiments which Newton men-

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[29] Herivel, *op. cit.*, p. 306 & p. 311.

[30] *Ibid.*, pp. 316–317 & p. 319.

[31] *Principles, op. cit.*, p. 1.

[32] *Ibid.*

tioned in his comments immediately following Definition I were already described in the above Definition 11 of the quantity of motion and this in connection with "the bulk of matter" which in effect implies inertial mass, it may now be very safely inferred that both Definition I and Definition II in *Principia* define the quantity of matter and the quantity of motion respectively as *quantity* in terms conjointly of density or velocity as *intensity* and bulk or quantity of matter as *extension* respectively. This interpretation of the present authors, they believe, certainly makes Newton's Definition I more intelligible and its historical situation clearer.