

The Formation of the Sommerfeld Quantum Theory of 1916

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I. Introduction

In 1916 Arnold Sommerfeld published a series of papers dealing with the quantum theory of spectra in *Annalen der Physik*,¹ in which the 1913 Bohr theory of the structure of the atom was greatly generalized and a general theory of relativistic Keplerian motion was developed. As I pointed out in my earlier paper,² the Bohr theory of 1913 was unable to determine the arrangement of electrons in the atom. At the earlier stage of development of the theory of atomic constitution in the 1910's, consideration of the chemical properties of elements played the cardinal part in unravelling the configuration of electrons in the atom. It was Sommerfeld's theory that first provided an actual starting-point for discussion of the structure of the atom on the basis of spectra. Sommerfeld thus brought about an important extension and refinement of the quantum theory of the atomic structure. On the basis of the Sommerfeld theory, the old quantum theory of spectra was developed, and this development led Bohr to formulate the correspondence principle, which in its turn opened the way to matrix mechanics. The aim of the present paper is to enquire into factors which contributed towards the formation of the Sommerfeld theory.

It was in 1911 that Sommerfeld first took up quantum theory. His first paper dealing with quantum theory was presented on January 7, 1911 to the Munich Academy of Science.³ Its subject was the theory of γ - and X-bremsstrahlung. In the same year he published two further papers, one of which was read at the 83rd meeting of German Scientists and Physicians on September 23,⁴ and the other at the first Solvay Congress held from October 30 to November 3.⁵

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¹ A. Sommerfeld, "Zur Quantentheorie der Spektrallinien, I. Theorie der Balmerischen Serie", *Ann. d. Phys.*, **51** (1916), 1-44; "II. Die Feinstruktur der Wasserstoff- und der wasserstoffähnlichen Linien", *Ibid.* (1916), 44-94; "III. Theorie der Röntgenspektren", *Ibid.* (1916), 125-167.

² S. Nisio, "The Role of the Chemical Considerations in the Development of Bohr Atom Model", *Jap. Stud. Hist. Sci.*, No. 6 (1967), 26-40.

³ A. Sommerfeld, "Über die Struktur der γ -Strahlen", *Sitzb. Bayer. Akad. Wiss.* (1911), 1-60.

⁴ A. Sommerfeld, "Das Plancksche Wirkungsquantum und seine allgemeine Bedeutung für die Molekularphysik", *Phys. ZS.*, **12** (1911), 1057-1069.

⁵ A. Sommerfeld, "Application de la théorie de l'élément d'action aux phénomènes moléculaires non périodiques", in P. Langevin et M. de Broglie (eds.), *La théorie du rayonnement et les quanta* (Paris, 1912), 313-392.

Both papers dealt with theory of bremsstrahlung, photo-electric effect, and ionization potential. The most remarkable thing in these 1911 papers is the quantum conditions employed in them. He assumed as the fundamental hypothesis relations of the forms,

$$\text{time} \times \text{energy} = h$$

and

$$\int_0^\tau (T - V) dt = \frac{h}{2\pi},$$

where T and V are the kinetic and potential energies of the electron and τ is the duration of the process considered. These conditions, especially the latter one, seem to suggest a close connection with the general quantum condition,

$$\int p dq = nh,$$

which was to be postulated by Sommerfeld in 1916. The conditions of 1911 appear as a first step towards the general quantum condition of 1916. It is therefore not surprising that historians of physics have regarded the 1911 papers as the first groping towards "a new method"⁶ or "a new mechanics".⁷ If we consider the fact that Planck, in his paper presented to the first Solvay Congress,⁸ proposed a quantum condition of the form

$$\int dq dp = h,$$

our impression is further strengthened. Was it not but a single step for Sommerfeld, by adapting Planck's idea to his theory, to reach the generalized quantum theory that he actually presented in 1916.

But, in fact, the difference between Sommerfeld's theories of 1911 and 1916 was not small. The nature of the problems dealt with by him in 1911 and 1916 was not the same. Whereas the theories in 1911 concerned an aperiodic system, the 1916 theory dealt with a periodic system. This alteration of the subject was brought about by the impact on Sommerfeld of Bohr's theory of 1913. His 1916 theory, however, was not merely a mathematical extension or refinement of the Bohr theory. Before he reached it, many new physical ideas had to be introduced, which also contributed to the development of Sommerfeld's interest and thought concerning quantum theory between the years 1911 and 1916. From where were those ideas supplied? To answer this question, it is necessary to examine to what extent and in what way the work of other people, especially

⁶ E. T. Whittaker, *A History of the Theories of Aether and Electricity, II, the Modern Theories* (London, 1953), 104-105.

⁷ M. Jammer, *The Conceptual Development of Quantum Mechanics* (New York, 1966), 54-55.

⁸ M. Planck, "La loi du rayonnement noir et l'hypothèse des quantités élémentaires d'action", *La théorie du rayonnement et les quanta*, 93-132, esp. 99.

of Bohr and Planck, influenced the development of Sommerfeld's idea during the period 1911 to 1916.⁹

In the following, I shall first survey, in section II, the general situation of the theory of quanta around the year 1911. Sommerfeld's concern with quanta before 1911, too, will be examined. Then in section III, I shall describe the main features of Sommerfeld's theoretical pursuits in 1911, paying attention to the supposed influence of Planck on him. In the period 1911 to 1915 he was far less productive in quantum theory, publishing only three papers, which, as will be seen in section IV, essentially presented little progress from the work in 1911. But in this very period Sommerfeld was following up Bohr's effort in developing his 1913 theory, and seems to have gained therefrom many physical ideas which were to be worked out mathematically in the 1916 papers. Section V is devoted to his debt to Bohr. Next, in section VI, Sommerfeld's 1916 theory is summarized in some detail as a preparation for the discussion in section VII of the nature of the quantum conditions employed by him in 1911 and 1916. This discussion will suggest the conclusion that Sommerfeld's 1916 theory may not be seen as the result of natural development of his 1911 theory. Intervention of the Bohr theory was indispensable for his accomplishments of 1916. In section VIII it is shown that part of the novel results in the 1916 paper stem from his exchange of opinions with Planck. In section IX, after looking briefly at the contemporary reactions of physicists to the Sommerfeld theory, I shall conclude the present paper with a consideration of the characteristic features of Sommerfeld's achievement in theoretical physics as developed from the following historical study of his quantum theory.

II. General Background Around 1911¹⁰

When in 1900 Planck derived his radiation formula,¹¹ he seems not to have attached cardinal significance to his assumption that the energy of an oscillator was an integral multiple of a unit $\epsilon = h\nu$. For him it was rather a purely mathe-

⁹ Neither Whittaker nor Jammer are concerned with this question in their rather detailed descriptions of the development of the quantum theory. In his recent book, Armin Hermann points out the probable influence of Planck on Sommerfeld's theory of 1911. However, he restricts his consideration to the period 1910-1912, and the development of Sommerfeld's theory after 1911 is left untouched. A. Hermann, *Frühgeschichte der Quantentheorie (1899-1913)* (Baden, 1969), esp. 118-139.

¹⁰ K. Amano, "Theory of Heat Radiation and the Origin of Quantum Theory" (in Japanese) *Selected papers* (Kyoto, 1948), 2, 1-63. O. A. Старосельская-Никитина и Л. С. Полак, "Макс Планк и Первый Сольвеевский Конгресс", in А. Ф. Иоффе и А. Т. Григорьян (ред.), *Макс Планк* (Москва, 1958), 221-243. R. McCormach, "Henri Poincaré and the Quantum Theory", *ISIS*, 58 (1967), 37-55. M. J. Klein, *Paul Ehrenfest I* (Amsterdam-London-New York, 1970), esp. 217-292. H. Kangro, *Vorgeschichte des Planckschen Strahlungsgesetzes* (Wiesbaden, 1970).

¹¹ M. Planck, "Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum", *Verh. d. Deutsch. Phys. Ges.*, 2 (1900), 237-245.

mathematical and formal assumption which was necessary in order to calculate the number of ways of dividing up total energy among oscillators. As Martin J. Klein and Hans Kangro have pointed out,¹³ Planck, in fact, emphasized the existence of the universal constant h which his radiation formula had revealed rather than the idea of a discrete energy unit. We may add that Planck applied the idea of a discrete energy unit only to the oscillator, not to free radiation.

The first to recognize the far-reaching implications of the Planck formula was A. Einstein. In 1905, he reached the conclusion that the energy quanta played a part not only in the interaction between radiation and matter, but also in the propagation of radiation through space.¹³ He indeed asserted that free radiation itself consisted of discrete light quanta. A year later Einstein extended the quantum structure of the oscillations of electromagnetic fields to the thermal oscillations of material molecules and thus developed a quantum theory of the specific heat of solid matter.¹⁴ Another proponent of the light quantum hypothesis was J. Stark. In 1907, he suggested the existence of an elementary quantum of energy and pointed out phenomena which in his opinion supported the quantum hypothesis; such as the photoelectric effect, the short wavelength limit of X-rays which depended on the kinetic energy of electrons, and others.¹⁵ Furthermore in 1909 he made an accurate observation of X-ray production by cathode rays, which appeared to him to confirm the light quantum hypothesis.¹⁶

Majority opinion in those days, however, was that the light quantum hypothesis was in absolute contradiction to the well-established electromagnetic theory of light. Many physicists adopted, instead of this radical view, the more conservative standpoint that the quantum discontinuities were rooted not in the radiation itself, but rather in some peculiar structure of the atom which emitted or absorbed the radiation. Among those who held such a view were A. Haas and J. J. Thomson. The latter, though he had once proposed the idea of a light particle,¹⁷ in a paper of 1910 dealt with the theory of radiation on the basis of the atomic model consisting of the electric doublet.¹⁸ There he asserts that "this

¹² M. Klein, *op. cit.* (note 10), 228-230. H. Kangro, *op. cit.* (note 10), 225-226.

¹³ A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt", *Ann. d. Phys.*, **17** (1905), 132-148.

¹⁴ A. Einstein, "Die Plancksche Theorie der Strahlung und die Theorie der Spezifischen Wärme", *Ann. d. Phys.*, **22** (1907), 180-190, 800.

¹⁵ J. Stark, "Elementarquantum der Energie, Modell der negativen und der positiven Elektrizität", *Phys. ZS.*, **8** (1907), 881-884.

¹⁶ J. Stark "Über Röntgenstrahlen und die atomistische Konstitution der Strahlung", *Phys. ZS.*, **10** (1909), 579-586. J. Stark, "Zur experimentellen Entscheidung zwischen Ätherwellen- und Lichtquanten-hypothese. I", *ibid.*, **10** (1909), 902-913.

¹⁷ J. J. Thomson, *Electricity and Matter* (New Haven, 1904), p. 62. J. J. Thomson, "On the Ionization of Gases by Ultra-Violet Light and on the Evidence as to the Structure of Light Afforded by its Electrical Effects", *Proc. Camb. Phil. Soc.*, **14** (1907), 417-424.

¹⁸ J. J. Thomson, "On the Theory of Radiation", *Phil. Mag.*, **20** (1910), 238-247.

theory enables us to explain the electrical effects produced by light, without assuming that light is made up of unalterable units, each containing a definite and, on Planck's hypothesis, a comparatively large amount of energy, a view which is exceedingly difficult to reconcile with well-known optical phenomena".¹⁹ Thomson intensified his attack on the light quantum hypothesis in his paper of 1912 saying that it was the mechanism of exchange of thermal and radiant energy and not the radiant energy itself which involved the element of discontinuity.²⁰ A. Haas, who was the first to attempt a quantum-theoretical treatment of the Thomson atom in 1910, considered the discreteness of energy to be derived from the structure of the atom.²¹

Planck, too, rejected such a radical view as the light quantum, but he did not attempt to explain h on the basis of the atomic structure.²² He rather wanted to regard h as connected with a fundamental principle of physics, the principle of least action. This view appeared for the first time in his widely read textbook *Vorlesungen über die Theorie der Wärmestrahlung*.²³ The relevant part of the book is section 149, which follows the section where the formula for the entropy of radiation is derived. Emphasizing the universal importance of the principle of least action, he called his universal constant h "the elementary quantum of action" or "the element of action" because of its having the dimension of energy \times time = action. Planck here also proposes the idea of quantizing the phase plane. He shows that in the phase plane the representative point of an oscillator of frequency ν with a given energy U describes an elliptical orbit, the area enclosed by this ellipse being just U/ν . Then he points out that the elementary cells or the elementary regions of equal probability corresponding to the energy interval ΔU are elliptical rings of area $(\Delta U/\nu)$, and that his 1900 relation $\epsilon = h\nu$ can be obtained by putting the energy element ΔU equal to ϵ and the area $(\Delta U/\nu)$ equal to the constant h , without the help of Wien's displacement law. Thus he asserts that the constant h has the meaning of the magnitude of an elementary region in the phase plane. His confidence in this view must have been strengthened by his study of the theory of relativity. In 1907 Planck developed the relativistic

¹⁹ J. J. Thomson, *Ibid.*, p. 246.

²⁰ J. J. Thomson, "The Unit Theory of Light", *Proc. Cam. Phil. Soc.*, 16 (1912), 643-652.

²¹ A. E. Haas, "Der Zusammenhang des Planckschen elementaren Wirkungsquantums mit den Grundgrößen der Elektronentheorie", *Jahrbuch d. Rad. u. El.*, 7 (1910), 261-268.

²² In the lectures delivered at Columbia University in April-May, 1909, Planck stated "J. J. Thomson inclines to the most radical view, as do J. Larmor, A. Einstein, and with him I. Stark, who even believes that the propagation of electromagnetic waves in a pure vacuum does not occur precisely in accordance with the Maxwellian field equations, but in definite energy quanta $h\nu$. I am of the opinion, on the other hand, that at present it is not necessary to proceed in so revolutionary a manner, and that one may come successfully through by seeking the significance of the energy quantum $h\nu$ solely in the mutual actions with which the resonators influence one another." M. Planck, *Acht Vorlesungen über Theoretische Physik* (Leipzig, 1910), 87-95, p. 95.

²³ M. Planck, *Vorlesungen über die Theorie der Wärmestrahlung* (Leipzig, 1906), 153-156.

dynamics of a moving system, and remarked that, since $H/\sqrt{c^2 - v^2}$, where H was the kinetic potential of the system concerned, was a relativistically invariant quantity, the action $W = \int_1^2 H dt$ would have the same value in any inertial coordinate system. Combining this fact with his view that the action had a definite elementary quantum h , he suggested that "every change in nature corresponds to a definite number of elements of action which is independent of the choice of the coordinate system."²⁴

Thus, from a quite early time Planck was inclined to consider h as a manifestation of a physical law connected with action. His view in this regard was probably rooted in his ideal of the pursuit of physics.²⁵ In his scientific autobiography, emphasizing that he has always looked upon the search for the absolute as the noblest and most worthwhile task of science, he writes: "The velocity of light is to the Theory of Relativity as the elementary quantum of action is to the Quantum Theory: it is its absolute core. In this connection, it turns out that a general principle of classical theory, the least-action principle, is also invariant with respect to the Theory of Relativity; accordingly, the quantum of action retains its significance in the Theory of Relativity as well".²⁶ The interpretation of h as the elementary quantum of action was repeatedly emphasized by Planck. In the paper presented to the first Solvay Congress in 1911, he pointed out that the hypothesis of a quantum of action would permit us to consider the elementary region of equal probability to be of finite magnitude $\int dp dq = h$, emphasizing that in this sense the hypothesis of quantum of action was more fundamental than that of energy quanta.²⁷

To sum up, until the year 1911 there had been proposed three different views as regards the physical meaning of the Planck constant h . The most radical one, which was held by Einstein, Stark, and a few others, was the view that the electromagnetic radiation had a particulate structure consisting of light quanta $h\nu$. The majority of physicists rejected this view. One of the remaining two views was to consider the energy quantum to be derived from a particular structure of the atom. The last was Planck's idea of interpreting h as the quantum of action representing the elementary region of equal probability in the phase space.

Sommerfeld, who had been a full professor of theoretical physics at Munich University since 1906, had not dealt with quantum theory until 1911. In the

²⁴ M. Planck, "Zur Dynamik bewegter Systeme", *Sitzb. Preuss. Akad. Wiss.* (1907), 542-570; *Ann. d. Phys.*, 26 (1908), 1-34; *Physikalische Abhandlungen und Vorträge* (Braunschweig, 1958), 2, 176-209. For information on this work of Planck's, see T. Hirose, "Relativistic Mechanics in Its Early Days-3" (in Japanese), *Buturigakusi Kenkyu* 6, No. 1 (1970), 27-61.

²⁵ M. Planck, *Ibid.*, *Phys. Abh.*, 2, 198. See H. Kangro, *op. cit.* (note 10), esp. 132.

²⁶ M. Planck, *Scientific Autobiography and Other Papers* (New York, 1968), 47-48.

²⁷ M. Planck, *op. cit.* (note 8), 113.

light of this fact Armin Hermann suggests that Sommerfeld had been sceptical over the quantum theory but in 1911 four factors induced him to draw his attention to the quantum theory.²⁸ But there seems to be no positive evidence that Sommerfeld had actually been sceptical over the quantum theory before 1911. In fact he had at times referred positively to the quantum theory. In a paper published in 1909 dealing with the theory of X-rays he writes that it is very probable that Planck's quantum of action plays a role in the case of emission and absorption of the fluorescent part of X-rays, *i.e.* the characteristic X-rays, by the atom.²⁹ In the same year, 1909, but prior to Sommerfeld, J. Stark published two papers dealing with the angular distribution of the intensity of X-ray bremsstrahlung.³⁰ He assumed that the bremsstrahlung, according to the classical electromagnetic theory, had equal intensity in all directions, and concluded that the experimental results could not be explained by the classical electromagnetic theory. He thus reaffirmed his previous view that electromagnetic radiation was of a particulate nature. Sommerfeld, in a letter to Stark dated December 4, 1909, criticized the latter's papers harshly because of an erroneous assumption of the angular distribution of the intensity of X-rays.³¹ He was so deeply convinced of his ability as a mathematical physicist that he was scornful of Stark's erroneous argument. He, however, says that he is by no means opposed to the quantum theory in general. In the same letter to Stark, he writes that he does not doubt the significance of the quantum of action. Sommerfeld, for his part, discussed in the 1909 paper cited above, the same problems as Stark on the basis of Maxwellian electromagnetic theory and confirmed that the general features of the experimental results could be deduced theoretically. Thus he concluded: "[by] the discussion above, our confidence in the validity of the electromagnetic theory for elementary processes in the electric field, which seems to have been undermined by the new speculation of a light quantum, is strengthened."³² Thus Sommerfeld, though admitting the general importance of the quantum theory, rejected the concept of the light quantum. In this respect, he was following the majority view of the day. To compare numerically the results of his calculations with the experimental results, however, it was necessary to know a parameter which was related to the atomic constant, such as the dura-

²⁸ The four factors pointed out by Hermann are: 1) The derivation of the Planck radiation formula by P. Debye, the then assistant of Sommerfeld, 2) W. Nernst's effort to organize the 1911 Solvay Congress, 3) H. A. Lorentz' lecture on the quantum theory in 1910 at Göttingen, 4) Sommerfeld's meeting with Einstein, supposedly in middle of 1910. A. Hermann, *op. cit.* (note 9), 122-125.

²⁹ A. Sommerfeld, "Über die Verteilung der Intensität bei der Emission von Röntgenstrahlen", *Phys. ZS.*, **10** (1909), 969-976, esp. 970.

³⁰ J. Stark, *op. cit.* (note 16).

³¹ A. Hermann, "Die frühe Diskussion zwischen Stark und Sommerfeld über die Quantenhypothese (1)", *Centaurus*, **12** (1967), 38-59, 45.

³² A. Sommerfeld, *op. cit.* (note 29), 976.

tion of bremsung of an electron or the distance through which acceleration of the electron occurred. But the electromagnetic theory alone could not suggest such a constant. For that purpose he had to introduce the Planck constant into his considerations. But this was not done until 1911.

III. Sommerfeld's Quantum Theory, 1911

Sommerfeld's approach in 1911 was novel in two respects. First he attempted to apply the quantum hypothesis, which had so far been applied exclusively to periodic elementary systems, to an aperiodic elementary process. Second, he proposed an expression of the quantum hypothesis different from those of Planck and Einstein. In doing so he reached a view opposite to the current one with regard to the relation of h and the atom. Contrary to the view held by many physicists that h might be derived from a particular structure of the atom, he supposed that the existence of the atom was a consequence of the existence of h . Sommerfeld's theory of 1911 was also novel in its approach, being different from all the methods so far proposed, which had been used exclusively for statistical mechanical problems. It must be borne in mind, however, that the basic idea of his theory might not have been entirely original. He, as we shall see, seems to have been profoundly influenced by Planck.

The theory of γ -rays which he presented at the Munich Academy of Science in January, 1911, as was the case in his earlier theory of X-rays, was based on the assumption that the γ -rays were emitted by bremsung of β -particles.³³ He considers γ -rays to be emitted when electrons are accelerated from rest to the speed with which they are ejected as β -rays from an atom. The electromagnetic theory of bremsstrahlung shows that the ratio of the energy of β -rays to that of γ -rays is

$$\frac{E_{\beta}}{E_{\gamma}} = \frac{6\pi c^2 l \sqrt{1 - \beta^2}}{e(e/m_0)\beta},$$

where l is the length of acceleration, that is, the range of the force, m_0 the rest mass of an electron, β the ratio v/c , v the velocity of a β -particle, and c the light velocity. To compare this expression with experimental data, it was necessary to determine the quantity l , the unknown parameter in the equation above. For this purpose he adopted "the fundamental hypothesis of Planck's radiation theory" and assumed "that one quantum of action h corresponds to each emission".³⁴ Taking, as the quantity of action, the product of the duration of acceleration τ and the total energy of the β -particle, he put

³³ A. Sommerfeld, *op. cit.* (note 3). It had generally been supposed that X-rays and γ -rays were analogous types of radiation. As to the nature of the γ -rays, as in the case of X-rays, there existed two theories, the corpuscular and pulse (bremsstrahlung) theories. See, for example, E. Rutherford, *Radioactive Substances and Their Radiations* (Cambridge, 1913), chap. VI, 286-292.

³⁴ A. Sommerfeld, *op. cit.* (note 3), 24-25.

$$\tau \cdot \frac{m_0 c^2}{\sqrt{1 - \beta^2}} = h,$$

from which l was obtained, the acceleration being assumed uniform. The energy ratio thus obtained turned out to agree, in order of magnitude, with that obtained from radioactivity studies. The value of l was shown to be $10^{-3} \sim 10^{-4}$ times smaller than the linear dimension of an atom, 10^{-8} cm.

In the same paper Sommerfeld also applied the same quantum condition to the discussion of continuous X-rays. Between the time interval τ , during which the cathode ray particle is decelerated to rest by the intra-atomic force, and the kinetic energy E_{kin} of the cathode ray particle, he postulated the relation,

$$\tau \cdot E_{kin} = \tau \cdot \frac{m_0 c^2}{\sqrt{1 - \beta^2}} (1 - \sqrt{1 - \beta^2}) = h.$$

The reason for taking E_{kin} instead of the total energy was that he supposed the atom to be capable of receiving only kinetic energy from the cathode ray particle. The result, in this case too, agreed with experimental results in order of magnitude. Sommerfeld then proceeded to show that, between the kinetic energy E_{kin} of the cathode ray particle and the frequency ν of X-rays produced by it, the same relation $E_{kin} = h\nu$ as that assumed by W. Wien and Stark in 1907 in their light quantum hypothesis,³⁵ could be derived from his hypothesis. He supposed that the hardness of X-rays or γ -rays was expressed in terms of the width λ of the pulse bremsstrahlung, λ being given by $\lambda = c\tau - l \cos \phi$, where ϕ is the angle measured from the direction of the incident electron. Since λ is equal to $c\tau$ when $\phi = \pi/2$, or, what amounts to the same thing, the average of λ is $c\tau$, if the width of pulse is assumed to correspond to the wavelength, Sommerfeld's fundamental relation $\tau \cdot E_{kin} = h$ corresponds to Wien and Stark's formula $E_{kin} = h\nu$. Thus he concluded that it was not necessary to discard the classical electromagnetic theory.³⁶

Thus Sommerfeld introduced the elementary quantum of action h in order to determine the linear dimension of the accelerating region or the duration of bremsung. However, no reference is made by Sommerfeld to Planck throughout this paper. This is probably because, by that time, Planck's hypothesis of quantum of action had become widely accepted by physicists, who had been acquainted with it through Planck's widely read textbook *Vorlesungen*. Sommerfeld, in his 1911 paper considered above, called h "quantum of action", which was a term used probably for the first time in Planck's *Vorlesungen*.³⁷

The next opportunities for Sommerfeld to mention the quantum of action

³⁵ J. Stark, *op. cit.* (note 15). W. Wien, "Über eine Berechnung der Wellenlänge der Röntgenstrahlen aus dem Planckschen Energieelement", *Göttinger Nachricht.* (1907), 598-601.

³⁶ A. Sommerfeld, *op. cit.* (note 3), 39-41. See also note 46, 710-711.

³⁷ M. Planck, *op. cit.* (note 23), 153-154.

were the annual meeting of German Scientists and Physicians in September, 1911 and the first Solvay Congress held about a month later. In the papers presented to these congresses,³⁸ he modified his former quantum condition so as to give it a more plausible appearance, and proposed the fundamental hypothesis that in every atomic process, namely in every elementary interaction between an atom and an electron, the atom emitted or absorbed a definite amount of action, the amount being determined by the condition,

$$\int_0^{\tau} (T - V) dt = \frac{h}{2\pi},$$

where τ is the duration of the interaction, and T and V are the kinetic and potential energies of the system considered.

Using this hypothesis he tried to explain the production of γ - and X-rays, the photoelectric effect, and the ionization potential. Applying this hypothesis to the theory of γ - and X-rays, he obtained results which, as in the previous paper, agreed with experimental results in order of magnitude. As for the photoelectric effect, Sommerfeld supposed that an electron quasi-elastically bound to the atom was liberated when, under the action of external radiation, the energy of the electron reached a certain value. The time interval τ which was required for the energy to reach this value was assumed to be determined by the fundamental quantum condition. He then showed that, in the case of resonance, when the frequency ν of incident radiation was equal to the eigenfrequency ν_0 of the bound electron, the relation $T = h\nu_0 = h\nu$, the Einstein photoelectric equation, could be derived. In the case without resonance, if $\nu > \nu_0$, the photoelectron is ejected, whereas if $\nu < \nu_0$, it cannot be ejected. To evaluate the ionization potential the fundamental condition was applied to an incident electron assuming that an atom was ionized by collision with an electron. The lowest velocity of the incident electron required for ionizing the atom, or the ionization potential, was obtained for a rare gas atom, for which V could be assumed approximately equal to zero. The value obtained agreed with the experimental results in order of magnitude.

The form of the quantum condition used in the September and November papers is different from the one used in the January paper. From the physical point of view, the former is more important than the latter. It seems likely that the influence of Planck lead Sommerfeld to modify his quantum condition. A. Hermann has pointed out the correspondence in 1911 between Sommerfeld and Planck which strongly suggests that Sommerfeld owed the modification to Planck.³⁹ Moreover, we can add other internal evidence which suggests Planck's influence upon Sommerfeld.

³⁸ A. Sommerfeld, *op. cit.* (notes 4 and 5).

³⁹ A. Hermann, *op. cit.* (note 9), 132-133.

In the papers of September and November, Sommerfeld gives following argument in order to justify his quantum condition. Since h has the dimension of action, he argues, it is natural to associate it with Hamilton's action principle. Now, the time integral of the relativistic Lagrangean H , the action, is invariant with respect to the Lorentz transformation. The absolute quantity in physics, therefore must be neither energy nor the time integral of energy, but action. It will then be reasonable, in the non-relativistic case too, to associate the universal constant h with the action. This reasoning based on a relativistic consideration resembles Planck's argument in his paper of 1907 on relativistic dynamics, which Sommerfeld cited in the places where he made the above argument.⁴⁰ As noted in the preceding section, Planck remarked that the action $W = \int_1^2 H dt$, being an invariant, would be associated with the quantum of action.

The resemblance of their arguments clearly indicates that Sommerfeld was considerably influenced by Planck's 1907 paper.⁴¹

I will now turn to the physical meaning which Sommerfeld then attached to the quantum hypothesis. As was noted at the beginning of this section, he considered the quantum of action h as the basis of the existence of the atom. Both at the meeting of German Scientists and the Solvay Congress, rejecting various attempts to explain h on the basis of atomic structure, he expressly declared: "From the opposite point of view I would like to consider the existence of molecules as a consequence of the existence of the elementary quantum of action, but not to attempt to deduce h from the molecular dimension".⁴² This view of h was really an innovation. But in other respects Sommerfeld remained rather conservative. He did not think that the quantum hypothesis would violate the Maxwellian electromagnetic theory. He considered his fundamental quantum condition to be compatible with the Maxwellian theory. In the paper presented to the Solvay Congress, he says that the electromagnetic theory of X-rays should be completed by the inclusion of the theory of radiation, that is, by introducing the element of action, and that both theories are complementary.⁴³ Being convinced of the general validity of the classical electromagnetic theory, he could regard the quantum hypothesis merely as a determining condition of the parameter concerning the nature of the atom, which could not be determined

⁴⁰ A. Sommerfeld, *op. cit.* (note 4), 1063; (note 5), 318-319.

⁴¹ Since the year 1907, when Sommerfeld was converted to a "relativist", he himself had pursued the relativity theory: see A. Hermann (ed.) *Albert Einstein/Arnold Sommerfeld Briefwechsel* (Basel, 1968), 20-21. Sommerfeld always paid due attention to all that was going on in various branches of physics: see H. Hermann, *op. cit.* (note 9), 123. Being so diligent a physicist, he would never have overlooked such a remarkable paper as Planck's, which played an important role in the development of relativistic dynamics.

⁴² A. Sommerfeld, *op. cit.* (note 4), 1066; (note 5), 363.

⁴³ A. Sommerfeld, *op. cit.* (note 5), 333.

solely by the electromagnetic theory. Thus the innovation brought about by him in the interpretation of h , that is, the view that the very existence of h was to determine the dimensions of the atom, was a result of his rather conservative view that the quantum of action did not destroy the electromagnetic theory but rather supplemented it.

If, as Sommerfeld believed, the acceptance of the quantum hypothesis does not imply the destruction of the classical electromagnetic theory, it would be natural to be sceptical as he was about the concept of the light quantum. In his conservative view of the quantum hypothesis as a whole, and his sceptical attitude to the light quantum in particular, Sommerfeld was apparently in line with Planck. In 1910, Planck manifestly rejected the theory of the light quantum by Einstein and Stark saying that such a theory would imply a retrogression over centuries in the theory of light.⁴⁴ He declared that in introducing the quantum of action one should be as conservative as possible. In the following two years, 1911 and 1912, he developed a modified theory of radiation assuming that the oscillator continuously absorbed electromagnetic energy whereas it discontinuously emitted the radiation.⁴⁵

Adherence to classical notions was common between Sommerfeld and Planck. In this connection, it is also worth noting that Sommerfeld's theory of photoelectric effect, in one respect, resembles Planck's modified radiation theory of 1911-12. In Sommerfeld's theory, an electron bound to an atom is assumed to absorb radiation energy continuously until its energy reaches a definite value which is determined by a quantum condition. The electron then is ejected abruptly. In Planck's theory, a resonator continuously absorbs radiation energy until its energy reaches the value $nh\nu$, when this energy is instantaneously emitted as radiation. The behaviour of the atom in the former theory and that of the resonator in the latter are similar to each other. Planck's theory was first presented to the German Physical Society in February, 1911. It is quite possible that Planck's theory furnished a hint to Sommerfeld. In fact in his paper dealing with the photoelectric effect in detail,⁴⁶ published in *Annalen der Physik* in 1913, Sommerfeld himself admits the similarity of his theory to Planck's, though, at the same time, he enumerates differences between their theories.

I said at the beginning of this section that one of the novelties in Som-

⁴⁴ M. Planck, "Zur Theorie der Wärmestrahlung", *Ann. d. Phys.*, **31** (1910), 758-768; *Phys. Abh.*, **2**, 237-247.

⁴⁵ M. Planck, "Eine neue Strahlungshypothese", *Verh. d. Deutsch. Phys. Ges.*, **13** (1911), 138-148; *Phys. Abh.*, **2**, 249-259. M. Planck, "Zur Hypothese der Quantenemission", *Sitzb. Preuss. Akad. Wiss.* (1911), 723-731; *Phys. Abh.*, **2**, 260-268. M. Planck, "Über die Begründung des Gesetzes der Schwarzen Strahlung", *Ann. d. Phys.*, **37** (1912), 642-656; *Phys. Abh.*, **2**, 287-301. For Planck's 1910-1911 work, see T. Hirose and S. Nisio, "The Genesis of the Bohr Atom and Planck's Theory of Radiation", *Jap. Stud. Hist. Sci.*, No. 9 (1970), 35-47, esp. 42-44.

⁴⁶ A. Sommerfeld u. P. Debye, "Theorie des lichtelektrischen Effektes vom Standpunkt des Wirkungsquantums", *Ann. d. Phys.*, **41** (1913), 873-930, esp. 923.

merfeld's theory was that he applied the notion of the quantum of action to aperiodic processes. Strictly speaking, however, this too was not entirely original with Sommerfeld, because it may be said that Planck had already suggested in his paper of 1907 the possibility of applying the quantum of action to aperiodic processes. In that paper Planck clearly states that "every change in nature corresponds to a definite number of elements of action."⁴⁷ From this statement one may easily draw the implication that an aperiodic process too would be governed, in one way or another, by the quantum of action.

It may therefore be said that Sommerfeld's contribution to the development of quantum theory in the year 1911 consists in his embodying Planck's recent ideas by performing detailed calculations on individual processes.

Before closing this section, we will glance at the reaction of contemporary physicists to Sommerfeld's theory. In the discussion following Sommerfeld's report at the Solvay Congress, Einstein characteristically remarked, referring to the calculation of the energy of X-rays, that a satisfactory agreement with experiment might not always be viewed as direct confirmation of the hypothesis on which the theory was based.⁴⁸ But other participants, especially Planck and Lorentz, were apparently in favour of Sommerfeld's theory of photoelectric effect.⁴⁹

Sommerfeld's theory in fact attracted the attention of several physicists for the reasons that it was an extension of the quantum theory to an aperiodic process, that it formulated a dynamical expression of the hypothesis of the quantum of action, and that it seemed compatible with the Maxwellian electromagnetic theory. In a survey article on the quantum theory, which was written in 1913 reviewing the proceedings of the first Solvay Congress, F. Reiche noted the novelty of Sommerfeld's theory, and posed the question of what relation existed between Sommerfeld's theory, which he said could be called the theory of the "elementary quantum of action", and Planck's and Einstein's theories.⁵⁰ Without giving answer of his own, Reiche remarked that the existence of alternative forms of the theory meant that the idea had not gelled and that it might take much time to reach a full understanding of the theory, but in view of the results derived from the quantum theory, he felt that the theory had achieved remarkable success in a short time. J. Franck and G. Hertz in their paper of 1913 dealing with the ionization potential of several gases, referred to Sommerfeld's theory, though they did not conclude that it was the best theory as regards agreement with their measurements.⁵¹ R. Seeliger in 1913 gave a survey of theoretical

⁴⁷ M. Planck, *op. cit.* (note 24), *Phys. Abh.*, 2, 198.

⁴⁸ A. Sommerfeld, *op. cit.* (note 5), 381.

⁴⁹ A. Sommerfeld, *op. cit.* (note 5), 391-392.

⁵⁰ F. Reiche, "Die Quantentheorie", *Naturwiss.*, 1 (1913), 549-553, 568-572.

⁵¹ J. Franck u. G. Hertz, "Messung der Ionisierungsspannung in verschiedener Gasen", *Verh. d. Deutsch. Phys. Ges.*, 15 (1913), 34-44.

and experimental research on the ionization of gases and praised the generality of Sommerfeld's theory.⁵² He remarked that Sommerfeld's theory considered the ionization process to be an energy transfer to an atom as a whole, whereas in other current theories the ionization potential was calculated on the basis of a particular atom model. Two years later in 1915, M. Wolfke in his paper "Über eine Verknüpfung des Sommerfeldschen Wirkungsprinzips mit der Planckschen Quantentheorie"⁵³ dealt with cavity radiation on the basis of Planck's general principle of 1907 that every change in nature corresponds to a definite number of elements of action. Assuming that this element of action was equal to Sommerfeld's, $h/2\pi$, he deduced that the energy of radiation with frequency ν was equal to an integral multiple of the energy quantum $h\nu$.

IV. Interlude, 1912-15

After the year 1911, and before 1915, Sommerfeld made no substantial progress in his pursuit of quantum theory, though during this period he published several papers on the quantum theory. I shall now look into the problems which were dealt with in these papers.

In 1913 Sommerfeld published two papers worth mentioning, one of which bore the title "Unsere gegenwärtigen Anschauungen über Röntgenstrahlung",⁵⁴ and the other, written with Debye, was entitled "Theorie des photoelektrischen Effekts vom Standpunkt des Wirkungsquantums".⁵⁵ In the former paper Sommerfeld reviewed the experimental and theoretical work published so far, and gave an account of his own theory of X-rays originally published in 1911. In the previous year, 1912, he made an important contribution to establishing the electromagnetic wave theory of X-rays by estimating the wavelength of X-rays at $\leq 4 \times 10^{-9}$ cm from an analysis of the X-ray diffraction which H. Haga and C. H. Wind in 1903, and B. Walter and R. Pohl in 1908 obtained by using slits.⁵⁶ In the 1913 paper Sommerfeld's emphasis was on the equivalence of his electromagnetic pulse theory and the wave theory. In the second paper of 1913, which dealt with the photoelectric effect, Sommerfeld discussed in detail, on the basis of his quantum hypothesis, the motion of an electron in an atom when the latter was exposed to light of various wavelengths. The results of this detailed calcula-

⁵² R. Seeliger, "Einige allgemeine Bemerkungen zur Theorie der Stossionisation", *Jahrb. d. Rad. u. Elektron.*, **10** (1913), 431-445.

⁵³ M. Wolfke, "Über eine Verknüpfung des Sommerfeldschen Wirkungsprinzips mit der Planckschen Quantentheorie", *Phys. ZS.*, **16** (1915), 207-209.

⁵⁴ A. Sommerfeld, "Unsere gegenwärtigen Anschauungen über Röntgenstrahlung", *Naturwiss.*, **1** (1913), 705-713.

⁵⁵ A. Sommerfeld u. P. Debye, *op. cit.* (note 46).

⁵⁶ A. Sommerfeld, "Über die Beugung der Röntgenstrahlen", *Ann. d. Phys.*, **38** (1912), 473-506. Sommerfeld had long studied problems concerning diffraction. For example, "Mathematische Theorie der Diffraction" (Habilitationsschrift), *Math. Ann.*, **47** (1896), 317-374; "Theoretisches über die Beugung der Röntgenstrahlen", *ZS. Math. u. Phys.*, **46** (1901), 11-97.

tion had already been presented at the two meetings of 1911, the meeting of the German Scientists and the Solvay Congress. The second 1913 paper, therefore, represents no essential progress in his theory.

For our account of Sommerfeld's work on the quantum theory in 1913 to be complete, the Wolfskehl lectures should also be mentioned.⁵⁷ This is a course of lectures on the kinetic theory of matter which were given by several authors at Göttingen University during the last week of April, 1913. Sommerfeld was among the lecturers. The others were Planck, Debye, Nernst, M. V. Smoluchowski, and Lorentz. Sommerfeld lectured on the kinetic theory of monoatomic gases based on the quantum theory. He attempted to explain the thermal properties of gases by supposing that, quite analogously to Debye's theory of solid matter, a number of Planck energy quanta were distributed over individual eigenfrequencies of "sound waves" in a cubical vessel with very hard walls containing a given number of monoatomic gas molecules. The contents of this lecture, however, as Sommerfeld himself remarked, were largely based on an investigation by his student W. Lenz.

In 1915 Sommerfeld published a theory of dispersion.⁵⁸ He derived the dispersion formula using Bohr's model of the molecule, which consisted of electrons rotating in rings around several nuclei distributed on the axis of symmetry. As the simplest examples he dealt with hydrogen, helium, nitrogen, and oxygen. In the first two cases the frequency of rotation of an electron determined by comparing his formula with experimental results agreed with that obtained by Bohr's angular momentum condition $p = h/2\pi$. In the latter cases, however, a considerable discrepancy was found. He was inclined to ascribe this discrepancy to the over-simplified model of the molecule used in his calculation. This theory of dispersion, however, was not entirely original to Sommerfeld, because, as he himself remarked, Debye had already obtained a quantitatively exact dispersion formula for hydrogen on the basis of Bohr's model of the hydrogen molecule.⁵⁹ His paper therefore was an extension of Debye's work, and may be characterized as a mathematical elaboration of it.

Having examined Sommerfeld's work concerning the quantum theory during the years 1912-1915, we may conclude that, judged from the published work, he made little essential progress in his investigations of quantum theory during this period. His papers published in this period were directed to refining the mathematical treatment of individual problems. The quantum hypothesis ap-

⁵⁷ A. Sommerfeld, "Probleme der freien Weglänge", in M. Planck, etc., *Vorträge Über Die Kinetische Theorie Der Materie Und Der Elektrizität* (Leipzig u. Berlin, 1914), 123-166.

⁵⁸ A. Sommerfeld, "Die allgemeine Dispersionsformel nach dem Bohrschen Modell", *Elster-Geitel-Festschrift* (Braunschweig, 1915), 549-584; *Gesammelte Schriften* (Braunschweig, 1968), 3, 136-171.

⁵⁹ P. Debye, "Die Konstitution des Wasserstoff-Moleküls", *Sitzb. Bayer. Akad. Wiss.* (1915), 1-26.

plied to them remained the same as that proposed in 1911. The last subject, the theory of dispersion, however, is remarkable in showing Sommerfeld's interest in the Bohr atom. In fact his interest in the Bohr theory had been evoked even before 1915 when, enlarging the Bohr theory, he first launched the generalized quantum theory of spectra. He had paid attention, since as early as 1913, to the development of Bohr's quantum theory. This is revealed by investigating his correspondence and the records of colloquia at his institute.

V. Debt to Bohr's Theoretical Pursuits

Sommerfeld was one of those who first reacted favorably to Bohr's theory. He alluded to Bohr's paper as early as July 1913 in a note appended to his Solvay Congress paper in the German version of the proceedings of the Congress.⁶⁰ There, in connection with the ionization potential of hydrogen, he refers to the relation between the binding energy and the radius of the hydrogen atom which Bohr gave in his first paper. But it was in his postcard to Bohr dated September 4, 1913, which L. Rosenfeld has published in his introduction to the reprint edition of Bohr's papers, that he praised the Bohr theory.⁶¹ There he writes that, having been pondering over the problem of expressing the Rydberg constant in terms of the Planck constant, he is extremely pleased with Bohr's success in solving this problem. At the same time, however, he says that he is, for the moment, sceptical about the atom model in general. Also he asks whether Bohr is going to look at the Zeeman effect on the basis of his model and adds that he himself wishes to attack this problem.

This postcard suggests that Sommerfeld paid attention to Bohr's work not because he was interested in the structure or the model of the atom, but rather because of his deep interest in spectra. He welcomed Bohr's theory primarily as a successful theory of the hydrogen spectrum. At the beginning of his 1915 paper on dispersion, which was discussed in the previous section, he writes "Die bedeutsamen Erfolge, welche das Bohrsche Atommodell für die Theorie der Spektrallinien gebracht hat".⁶²

Sommerfeld had been greatly interested in spectroscopy, especially in the

⁶⁰ A. Sommerfeld, "Die Bedeutung des Wirkungsquantums für unperiodische Molekularprozesse in der Physik", *Abh. d. deutsch. Bunsenges. f. angewandte phys. Chemie* (Halle, 1914), 7, 252-317, 299; *Gesammelte Schriften*, 3, 70-135, 117.

⁶¹ L. Rosenfeld, "Introduction" to N. Bohr, *On the Constitution of Atoms and Molecules* (Copenhagen, 1963), L11.

⁶² A. Sommerfeld, *op. cit.* (note 58), *Gesammelte Schriften*, 3, 136. In a memoir written in 1942 Sommerfeld says that he had been interested in the problem of whether there was a relation between Rydberg's constant and Planck's constant and that Bohr's 1913 paper made deep impression on him. He also writes that he had been interested in the anomalous Zeeman effect, about which he had been told by his friend, mathematician C. Runge who, with H. Kayser, had been measuring it. A. Sommerfeld, "Zwanzig Jahre Spektroskopischer Theorie in München", *Scientia*, 1942, 123-130; *Gesammelte Schriften*, 4, 632-639, esp. 633-634.

Zeeman effect for some time. For example, in 1913 he published a paper dealing with the Zeeman effect of an anisotropically bound electron and the Paschen-Back effect.⁶³ Then, in 1914, he published a paper dealing with Voigt's theory of Zeeman effect.⁶⁴ His deep interest in spectroscopy is reflected also in the topics discussed at his weekly colloquium, known as the Munich Wednesday colloquium.⁶⁵ There from December 1912 to January 1913, N. Weber reported on Stark's discussion of the regularity of spectra in his *Atomdynamik*, Vol. 2, and Sommerfeld talked about his own theoretical consideration of the Paschen-Back effect. The discovery of the Stark effect, which was first reported by Stark on November 20, 1913 at the meeting of the Berlin Academy of Science, aroused Sommerfeld's keen interest. As early as December 10, 1913, at the Munich colloquium, E. Wagner commented on Stark's paper and demonstrated a Stark effect experiment. From January of 1914 on, Bohr's work was constantly taken up at the Munich colloquium.

The sequence of these dates seems to suggest that in fact Sommerfeld's chief reason for considering the Bohr atom was the discovery of the Stark effect. And at the same time, interest in the structure of the atom also seems to have been evoked in Sommerfeld. Before the Bohr theory of 1913 was published, no problem concerning atomic structure had been taken up as a topic at the Munich colloquium. As his postcard to Bohr clearly shows, in those days Sommerfeld was not interested in the real structure of the individual atom. The fact that, though in January 1911, when discussing the theory of γ -rays, he obtained for the order of magnitude of the length of *bremsung* the value $l \sim 10^{-22}$ cm,⁶⁶ he did not pay even the slightest attention to Rutherford's nucleus, whose dimensions had been shown to be of the same order as Sommerfeld's l , seems to show his indifference to the structure of the atom. In his theory of the photoelectric effect of 1913 Sommerfeld postulated an atom in which an electron was quasi-elastically bound. However, regarding it "as a substitute for the atom model, or so to speak an analytical model of the main character of the atom, which, because of its generality and modesty, was probably more trustworthy than any special constitution of the atom by mass, charge, and motion",⁶⁷ he abandoned the attempt to deduce the eigenfrequencies of electron vibration in the atom.

⁶³ A. Sommerfeld, "Der Zeemaneffekt eines anisotrop gebundenen Elektrons und die Beobachtungen von Paschen-Back", *Ann. d. Phys.*, **40** (1913), 748-774.

⁶⁴ A. Sommerfeld, "Zur Voigtschen Theorie des Zeeman-Effektes", *Göttinger Nachricht.* (1914), 207-229; *Gesammelte Schriften*, **3**, 47-69.

⁶⁵ Register volume for Münchener physikalisches Mittwochs-Colloquium, Dec., 1908 to May, 1939, in the *Archive for the History of Quantum Physics* deposited at the Library of the American Philosophical Society in Philadelphia, and other places.

⁶⁶ A. Sommerfeld, *op. cit.* (note 3), 29.

⁶⁷ A. Sommerfeld u. P. Debye, *op. cit.* (note 46), 923.

Generally speaking, it seems that in Germany problems concerning the real structure of the atom received little interest at that time. For instance, Planck's resonator was considered by himself as merely a mathematical model from which the radiation formula could be derived. In his letter to W. H. Bragg dated December 20, 1911, Rutherford described his impression of the first Solvay Congress saying: "I was rather struck in Brussels by the fact that continental people do not seem to be in the least interested to form a physical idea of the basis of Planck's theory. They are quite content to explain everything on a certain assumption, and do not worry their heads about the real cause of the thing. I must, I think, say that the English point of view is much more physical and much to be preferred".⁶⁸ Bohr's theory of atomic constitution was, as I, with T. Hirose, have emphasized in an earlier paper, initiated by his deep concern for an explanation of the chemical properties of atoms in terms of their structure.⁶⁹ Bohr may possibly have acquired the English taste for the structure of the atom through his contact with J. J. Thomson and Rutherford. By inheriting through Bohr the English concern over the structure of the atom, which was alien to the German tradition, Sommerfeld now was able to develop a theory of spectra, and to use it as an important clue to atomic structure.⁷⁰ From the above considerations it may be concluded that an important aspect of the impact of Bohr's theory on Sommerfeld was that it aroused in him interest in problems concerning the structure of the atom, and thus initiated his theoretical inquiry into the structure of the atom.

His efforts in this direction culminate in his generalized quantum theory of 1916. Among the achievements of this theory are the relativistic explanation of the fine structure of spectral lines and the general treatment of Keplerian motion. The initial drive for these achievement too, however, as I will show below was owed by Sommerfeld to Bohr. Early in 1914 Bohr, while he was involved in a dispute concerning the assignment of spectral lines of hydrogen and helium,⁷¹ attempted to explain the Zeeman and Stark effects on the basis of his theory.⁷² Bohr supposed that each series of the spectrum of an element corresponded to a series of stationary states of the atom in which one of the electrons moved outside the rest. The configuration of the inner electrons was assumed to be nearly the same in each series, while that of the outer electron changed from one state to another roughly in the same way as the electron in the stationary

⁶⁸ A. S. Eve, *Rutherford* (Cambridge, 1939), 208.

⁶⁹ T. Hirose and S. Nisio, "Formation of Bohr's Theory of Atomic Constitution", *Jap. Stud. Hist. Sci.*, No. 3 (1964), 6-28.

⁷⁰ The role of the English tradition was suggested to the author by Prof. Tetu Hirose.

⁷¹ N. Bohr, "The Spectra of Helium and Hydrogen", *Nature*, **92** (1913), 231-233; "The Spectra of Hydrogen and Helium", *Nature*, **95** (1915), 6-7.

⁷² N. Bohr, "On the Effect of Electric and Magnetic Fields on Spectral Lines", *Phil. Mag.*, **27** (1914), 506-524.

states of the hydrogen atom. Bohr further assumed that the doublets in the spectra of many elements were due to small perturbing forces originating in the configuration of the inner electrons, which exerted different effects on the motion of the outer electron according to the location of the latter's orbit. In the case of the hydrogen spectrum, however, Bohr was forced to devise a different mechanism because the lines observed were diffuse and the separation of the components varied widely according to the observers. He supposed that the effect was due to the electric field causing the gaseous discharge.

Meanwhile, W. E. Curtis, performing very accurate measurements of the wavelength of the hydrogen lines, found a small deviation from the Balmer formula.⁷³ On the basis of this result he concluded that Bohr's argument on the doublet lines H_α and H_β observed in the Balmer series could not be maintained. He admitted that Bohr's interpretation might account for the fact that the separation of the doublet lines varied according to the experimenter, because the electric fields used by different observers were usually different, but this would not be able to explain, he pointed out, why the ratio of the separations of H_α and H_β doublet components had varied according to a single observer.

Searching for a way to overcome this problem, Bohr early in 1915 hit upon the variation of the electron mass with its velocity.⁷⁴ He thought that the deviation of the hydrogen spectrum from the Balmer formula, observed by Curtis, might be accounted for by this relativistic effect. Assuming a circular orbit for the electron, he obtained the formula for frequency with the correction:

$$\nu = \frac{2\pi^2 e^4 m M}{h^3 (m + M)} \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\} \left[1 + \frac{\pi^2 e^4}{c^2 h^2} \left\{ \frac{1}{n_1^2} + \frac{1}{n_2^2} \right\} \right],$$

where m was the mass of the electron and M the mass of the nucleus. Here the motion of the nucleus too is taken into account. However, this correction could account for only one-third of the deviation observed by Curtis. Bohr however thought that since the separation of the components of each hydrogen doublet was much greater than the deviation from the Balmer formula, it was of primary importance to explain the doublets theoretically. Bohr felt that the cause of the doublet might be that electron orbits were not circular. If the orbit was not circular, he reasoned, it would rotate around an axis through the nucleus when the velocity of the electron was high. The frequency of the rotation of the orbit would depend on its eccentricity. For very small eccentricity the ratio between the frequencies of rotation of the orbit and of revolution of the electron was given by $2\pi^2 e^4 / n^2 c^2 h^2$, which, when $n = 2$, was of the same order as the ob-

⁷³ W. E. Curtis, "Wave-lengths of Hydrogen Lines and Determination of the Series Constant", *Proc. Roy. Soc.*, **90A** (1914), 605-620.

⁷⁴ N. Bohr, "On the Series Spectrum of Hydrogen and the Structure of the Atom", *Phil. Mag.*, **29** (1915), 332-335.

served separation of the doublet lines of hydrogen. Having obtained this result, however, Bohr remarked that in view of the many assumptions made for the calculation, it would be of little use to pursue this problem in further detail before more accurate measurements of the separation of doublets were possible.

Sommerfeld is generally understood to have been the first to attempt to account for the hydrogen doublet by considering the effect of the relativistic variation of electron mass. But our examination above clearly shows that the attempt was first made by Bohr, early in 1915.

The initial step to extend the Bohr theory of 1913 to Keplerian motion was also taken in Bohr's early-1914 paper examined above.⁷⁵ Searching for an explanation of the Stark effect, he suggested the existence of elliptical orbits in the hydrogen atom, arguing as follows: as a result of the influence of an external electric field, the electron orbit, which was originally circular, will be deformed. If the force is not perpendicular to the plane of the orbit, the deformation will become considerable in the course of time. The orbit may at every moment be considered an ellipse with the nucleus as one of the foci. Due to the effect of the field, the direction of the major axis as well as the eccentricity of the ellipse will gradually vary. Though detailed investigation of the motion of the electron may become enormously complicated in such a case, it is easily shown that there are two stationary orbits of the electron, whose eccentricities are both equal to 1 and whose major axes lie parallel to the axis of the external field; the orbits simply consist of straight lines through the nucleus, one parallel to the axis of the field, another antiparallel to it. By means of a correspondence consideration, Bohr concluded the energy A_n of the n -th state for these orbits to be

$$A_n = -\frac{2\pi^2 e^4 m}{n^2 \hbar^2} \left(1 \pm E \frac{3\hbar^4 n^4}{16\pi^4 e^5 m^2} \right),$$

where E is the intensity of the external field. The principal term of the right-hand side represents the energy when the external field is absent, and has been proved to be exact for any value of n . The whole expression above for A_n , therefore, may be assumed to hold good for any n . Then it follows that the hydrogen atom in an external field does have two series of stationary states. Assuming that the system can transit only between different states in each series, the frequency of the radiation emitted by a transition between two states corresponding to n_2 and n_1 is obtained as

$$\nu = \frac{1}{\hbar} (A_{n_2} - A_{n_1}) = \frac{2\pi^2 e^4 m}{\hbar^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \left(1 \mp E \frac{3\hbar^4}{16\pi^4 e^5 m^2} n_1^2 n_2^2 \right).$$

This formula gives for the separation of any hydrogen doublet the value

$$\Delta\nu = \frac{3}{4\pi^2} \frac{\hbar}{em} E(n_2^2 - n_1^2),$$

⁷⁵ N. Bohr, *op. cit.* (note 72), 513-517.

which is proportional to the electric field. In the way sketched out above, Bohr gave an explanation of the general features of the Stark effect in the hydrogen spectrum. Discussing the extension of the theory to other general elements, Bohr remarked on the special features of the case of hydrogen. In this case, the external field acting upon the stationary state has been considered to cause variation in the orientation and eccentricity of the orbit of the electron in the presence of the field. The possibility of such a variation is clear from the fact that in the absence of an external field every elliptical orbit is stationary. But in the case of other elements, Bohr argued, there will be perturbing forces exerted by the inner electrons and the condition just mentioned is not satisfied. The effect of an external electric field on the stationary states, therefore, is expected to be much smaller than for the corresponding states of the hydrogen atom.

Thus it is clear that Bohr had already suggested the existence of elliptical orbits of the electron in the hydrogen atom. But his explanation of the Stark effect itself, though it was more advanced than the first attempt by E. Warburg in December 1913 to explain the Stark effect on the basis of the Bohr atom,⁷⁶ was by no means complete. It could explain only the main feature of the two strongest components (π -components) of the split hydrogen lines. This must have been a challenge to Sommerfeld, who considered it the most important area of research of physics that the results of detailed theoretical calculation should agree with experiment.⁷⁷ On May 27, 1914, in fact, he took up Bohr's theory of the Stark effect as the subject of the Munich colloquium. As will shortly be discussed, the difficulty in explaining the Stark effect seems to be the reason why his general theory was not published until the end of 1915.

Not only the theory of the Stark effect, but the whole theoretical pursuit of Bohr was followed up at the Munich colloquium.⁷⁸ On November 19, 1913, P. P. Ewald made a report on the annual meeting of the British Association held in September, where Bohr's 1913 theory was for the first time discussed publicly. However, emphasis in Ewald's report was laid upon Bragg's work on X-ray diffraction presented at the meeting. On January 26, 1914, P. S. Epstein reported on the work of Bohr published during 1913-14 in *Philosophical Magazine*. On May 27, 1914, as mentioned above, Sommerfeld and Lenz reported on Bohr's

⁷⁶ E. Warburg, "Bemerkungen zu der Aufspaltung der Spektrallinien im elektrischen Feld", *Verh. Deutsch. Phys. Ges.*, 15 (1913), 1259-1266. Warburg presented this paper at the meeting of the German Physical Society on December 5, 1913, shortly after the discovery by Stark, which was reported at the meeting of the Berlin Academy of Science on November 20, 1913: J. Stark, "Beobachtungen über den Effekt des elektrischen Feldes auf Spektrallinien", *Sitzb. Preuss. Akad. Wiss.* (1913), 932-946.

⁷⁷ For example, he wrote in Part I of the *Annalen* paper (*op. cit.* note 1) that his generalized quantum equations expressed for each degree of freedom were inspired by experimental results of the Balmer series. See also section VI and VIII.

⁷⁸ Register volume for Münchener physikalisches Mittwochs-Colloquium, *op. cit.* (note 65).

theory of the Stark effect in *Philosophical Magazine* (1914) and as well as on W. Voigt's theory of the Stark effect in *Göttingen Nachrichten*. Subsequently, on July 15 Bohr himself gave a guest lecture on his atom model, especially on the spectra of helium and hydrogen, and on November 27, 1915, Sommerfeld talked on the recent work of Bohr.⁷⁹

Thus in 1914, while investigating the dispersion theory by means of Bohr's model of the atom and the molecule, which was to be published in 1915, Sommerfeld began to develop an elaborate theory of the Balmer series. At the beginning of the paper presented on December 6, 1915, he declared that about a year earlier he delivered a lecture on the subject to be discussed in that paper.⁸⁰ He wrote also in his autobiographical sketch that he had talked about his extension of Bohr's theory to his colleagues in the winter of 1914/15.⁸¹ The reason for delaying publication was, according to Sommerfeld, that his theory could not satisfactorily explain the Stark effect.⁸² It was early in 1916 that Epstein, a pupil of Sommerfeld and K. Schwarzschild, director of the Astrophysical Observatory at Potsdam, succeeded independently in explaining the Stark effect on the Balmer series.⁸³ As to the Zeeman effect, Sommerfeld himself gave its explanation in 1916.⁸⁴

From the above considerations, it follows that it was probably because Sommerfeld was spending time to follow up the development of Bohr's ideas after 1913 that the extension of the Bohr theory had not been effected until the end of 1915. After all, the necessary ideas such as the existence of elliptical orbits of the electron in the hydrogen atom and relativistic effects as the cause of the doublet of the hydrogen spectrum had been advanced by Bohr during 1914-15, Sommerfeld worked them out mathematically into a complete theory. Even more fundamentally, as was noted earlier, his interest in problems concerning the structure of the atom first arose due to Bohr's influence.

VI. Sommerfeld's Quantum Theory, 1915-1916

The discussion in the previous section shows that Sommerfeld's quantum

⁷⁹ This work is probably N. Bohr, "On the Quantum Theory of Radiation and the Structure of the Atom", *Phil. Mag.*, **30** (1915), 394-415.

⁸⁰ A. Sommerfeld, "Zur Theorie der Balmerschen Serie", *Sitzb. Bayer. Akad. Wiss.* (1915), 425-458, 426.

⁸¹ A. Sommerfeld, "Autobiographische Skizze", Herausgegeben von F. Bopp. in *Geist und Gestalt. Biographische Beiträge zur Geschichte der Bayerischen Akademie etc.*, **2** (München, 1959), 100-109, 105; *Gesammelte Schriften*, **4**, 673-682, 678.

⁸² A. Sommerfeld, *op. cit.* (note 80), 426.

⁸³ P. S. Epstein, "Zur Theorie des Starkeffektes", *Ann. d. Phys.*, **50** (1916), 489-520, received on May 9, 1916. The preliminary report was presented on March 26, 1916 to *Phys. ZS.*: P. S. Epstein, "Zur Theorie des Starkeffekts", *Phys. ZS.*, **17** (1916), 148-150. K. Schwarzschild, "Zur Quantenhypothese", *Sitzb. Preuss. Akad. Wiss.* (1916), 548-568.

⁸⁴ A. Sommerfeld, "Zur Theorie des Zeeman-Effekts der Wasserstofflinien, mit einem Anhang über den Stark-Effekt", *Phys. ZS.*, **17** (1916), 491-507.

theory of 1916 may be called a generalization of the Bohr theory of atomic constitution. At the same time, however, we should not overlook the debt that he owed to Planck. There is more than one aspect of the mutual connection between Sommerfeld and Planck in their pursuit of quantum theory. But before considering this, we will examine the essential results obtained by Sommerfeld in his papers of 1915–16.

The first report of Sommerfeld's 1916 theory was in fact presented on December 6, 1915 to the Munich Academy of Science, and was soon published.⁸⁵ This work was, as its title "Theorie der Balmerschen Serie" suggested, an extension of Bohr's theory of hydrogen spectra. Here, in contrast to his earlier theory of 1911 which dealt with aperiodic elementary processes, he turns his attention to the periodic process and, correspondingly, he replaces his former fundamental hypothesis by the quantization of phase space: $\iint dp dq = h$ or its more convenient form $\int p dq = nh$. From this condition can be derived the quantum conditions which have so far been supposed for the motions of a periodic system with one degree of freedom, such as the rotation of a gas molecule and the orbital motion of the Bohr atom. He then considers the application of the fundamental hypothesis to a more general model of the hydrogen atom in which the electron is permitted to describe not only circular orbits, but also elliptical orbits. In this case, the quantum condition $\int p_\phi d\phi = nh$ for the angle ϕ and corresponding angular momentum p_ϕ , together with the Bohr frequency condition, gave an expression for the frequencies of hydrogen spectral lines which varies continuously with the continuous change of eccentricity. This result obviously conflicts with the Balmer formula. Sommerfeld therefore supposes that it is necessary to add another condition to quantize the eccentricity of the elliptical orbits. The simplest assumption is that the radial variables should be subjected to, by analogy with the angular variables, the condition,

$$\int p_r dr = n'h,$$

which guarantees a reasonable expression for the frequency:

$$\nu = N \left(\frac{1}{(n + n')^2} - \frac{1}{(m + m')^2} \right),$$

where N is Rydberg's constant. With these quantum conditions Sommerfeld could also deduce the relation $b/a = n/(n + n')$ which determines the ratio of the minor semiaxis b to the major semiaxis a of the ellipse. This result shows, Sommerfeld thought, that a line of the Balmer series should be a superposition

⁸⁵ A. Sommerfeld, *op. cit.* (note 80).

of several lines corresponding to various electron transition processes between orbits with varied eccentricity. He therefore supposes that there must be a rule, a selection rule as we might call it, that determines which of the possible transitions would really occur, and discusses possible forms of the rule at some length.

As regards the spectral lines of elements other than hydrogen, Sommerfeld, like Bohr, supposes that an analogy with the case of hydrogen might be applicable, except that the term $1/(n + n')^2$ appearing in the expression for the frequencies of hydrogen lines should be replaced by a function of n and n' , $\phi(n, n')$. The number of spectral series and the multiple lines observed in the spectra of elements other than hydrogen may be explained by supposing the existence of a series of such stationary states of the atom in which one of the electrons moves outside the rest. He argues that the last inference was supported by the simplest case of hydrogen. He also suggests that in the general case, besides radial and angular variables r and ϕ , the z component should also be taken into consideration, the frequencies being expressed in terms of a function $\phi(n, n', n'')$.

In the next paper presented on January 8, 1916 to the Munich Academy⁸⁶ Sommerfeld attempted to verify the theory outlined above, in particular the existence of the elliptical orbits, by considering the fine structure of hydrogen lines on the basis of that theory combined with the relativistic variation of electron mass. In this paper, detailed calculations are carried out concerning the number of components of the fine structure, the interval between the components, and their intensities in a number of individual cases. Comparison of the result of the calculation with experimental observations is made as far as possible for the spectra of hydrogen, ionized and neutral helium, lithium, and especially for the K- and L-series of the characteristic X-ray spectra which Sommerfeld supposes to show a marked relativistic effect. He notices disagreements between his calculations and experimental observations on the splitting of the doublet of hydrogen, and on the dependence of the splitting of the L-doublet upon the atomic number. However, on February 10, 1916 he added an appendix to the January 1916 paper, in which he showed that these disagreements were removed if the region of the phase integral for the azimuthal variable ϕ was corrected:⁸⁷ he had formerly taken as the limits of the integral the azimuth ($\phi = 0$) of the first perihelion and that of the next perihelion (ϕ was not equal to 2π due to the relativistic correction). But he now replaced them by the region from $\phi = 0$ to $\phi = 2\pi$, as in the case of non-relativistic motion.

It was in the three-part article published in *Annalen der Physik* about seven

⁸⁶ A. Sommerfeld, "Die Feinstruktur der Wasserstoff- und der Wasserstoff-ähnlichen Linien", *op. cit.* (note 80), 459-500. For a detailed account of the relativistic theory of fine structure, see, for example, M. Born, *The Mechanics of the Atom* (New York, 1960), §33, 201-207.

⁸⁷ A. Sommerfeld, *Ibid.*, 498-500.

months later that Sommerfeld's quantum theory of a system with many degrees of freedom was fully developed.⁸⁸ The major part of this article may be considered essentially a reproduction of the previous two papers of December, 1915 and January, 1916. Part I and Part II of this article correspond, respectively, to the first and the second of these papers. The theory of the fine structure of the characteristic X-ray spectra, previously contained in the second paper, is now enlarged to be a separate Part III. There are, however, many essential revisions and additions throughout the article.

Part I of the article deals with the theory of the Balmer series.⁸⁹ Sommerfeld intends to generalize Bohr's theory of the hydrogen spectra. For this purpose he first postulates a generalized quantum hypothesis for a system with many degrees of freedom:

$$\int dp_i dq_i = h, \quad i = 1, 2, \dots, f \quad (f: \text{the degree of freedom}),$$

or in the more convenient form

$$\int p_{n_i} dq_i = n_i h, \quad \begin{aligned} n &= 1, 2, \dots \\ i &= 1, 2, \dots, f. \end{aligned}$$

Sommerfeld applies his quantum condition to simple examples such as the linear oscillation of a mass point, the rotation of a mass point with a constant angular velocity, and the behaviour of the angular coordinate ϕ in Keplerian motion. In the case of Keplerian motion, as discussed in the December 1915 paper, the quantum conditions for the radial coordinate r , as well as for the angular variable ϕ , together with the Bohr frequency condition, give for the Balmer series the formula

$$\nu = N \left(\frac{1}{(n + n')^2} - \frac{1}{(m + m')^2} \right).$$

Then Sommerfeld discusses the number and the form of quantized orbits with a given energy and attempts, as in the previous paper, to set up a selection rule, that is, the relation, the quantum inequality as he calls it, between the quantum numbers n, n', m, m' , which determines which of the possible transitions between various quantized orbits may really occur. On this basis he estimates the relative intensity of the components of hydrogen doublets.

Sommerfeld then poses the question whether or not the orbits are spatially quantized. To answer this question, it is necessary to fix one direction in space as the axis of reference. He supposes that an external electric or magnetic field or the electron configuration of inner core of the atom would provide the axis of reference. The intensity of these external or internal agents then has to be

⁸⁸ A. Sommerfeld, *op. cit.* (note 1).

⁸⁹ A. Sommerfeld, *op. cit.* (note 1), I, 1-44.

reduced to zero so that only the privileged direction of orientation of the atom would remain. This direction is taken as the axis of the equatorial plane of the polar coordinate system r, θ, ϕ . Here r denotes the radius vector, θ the latitude, and ϕ the azimuth. The fundamental assumption gives as the quantum conditions for three variables:

$$\int_0^{2\pi} p_\phi d\phi = n_1 h, \quad \int p_\theta d\theta = n_2 h, \quad \int p_r dr = n' h.$$

Relations between these quantum numbers and the inclination α of the orbital plane with respect to the equatorial plane are obtained as

$$\cos \alpha = \frac{n_1}{n_1 + n_2}, \quad 2\pi p = (n_1 + n_2) h.$$

This first equation shows that the inclination is quantized. The space quantization shows that to each line of the Balmer series there corresponds more than one kind of transition. These complex lines would, Sommerfeld supposes, be separated into their individual components corresponding to different n_1 's and n_2 's by the Zeeman and the Stark effects.

In the last section of Part I, Sommerfeld discusses the choice of a coordinate system in dealing with the dynamics of the atom. A similar problem had already been discussed in the corresponding last section of the previous December 1915 paper entitled "Über die Unabhängigkeit des Quantenansatzes von der Wahl der Koordinaten. Beziehungen zur allgemeinen Mechanik".⁹⁰ Referring to the fundamental hypothesis in terms of the quantum of action proposed in his former 1911 theory, Sommerfeld argued there as follows: According to general dynamics, the kinetic energy T is written as $T = \frac{1}{2} \sum p \dot{q}$. In the case of periodic or quasi-periodic motion, the time average of the kinetic energy \bar{T} is given by

$$\bar{T} = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} T dt = \frac{1}{2\tau} \sum \int p dq = \frac{1}{2\tau} (n + n' + \dots) h,$$

where τ is the period. From this equation it is readily seen that the sum of quantum numbers, $(n + n' + \dots)$, has a definite physical meaning independent of the choice of coordinate systems, but it is undecided whether each quantum number always corresponds to individual coordinates separately. This is the case for Keplerian motion, the coordinates ϕ and z being cyclic. Both ϕ and z then have a dynamical meaning and their phase integrals are constant, being equal to $2\pi p$ and 0, respectively. When the mean kinetic energy \bar{T} of the system is equal to the mean potential energy \bar{V} , the total energy W becomes equal to an integral multiple of $h\nu$. In the case that the electron is subjected to an arbitrary central force $F = Kr^{\epsilon-1}r$, the mean kinetic energy \bar{T} is given by

⁹⁰ A. Sommerfeld, *op. cit.* (note 80), 454–458.

$$\bar{T} = \frac{\kappa + 1}{\kappa + 3} W.$$

By comparing this equation with the equation above $2\bar{T}r = (n + n' + \dots)h$, the relation between W and h , which resembles the condition, energy \times time = h , proposed in the first 1911 paper on γ - and X-rays, is obtained.

Now in the corresponding section of Part I of the *Annalen* paper, the argument outlined above was replaced by a consideration of theories of the Stark effect which had just been presented by Epstein and Schwarzschild, independently of each other. Epstein sent his preliminary report to *Phys. ZS.* on March 26, 1916 and subsequently gave a detailed account in a paper in *Annalen*.⁹¹ Schwarzschild read his paper at the meeting of the German Physical Society on March 30, 1916.⁹² In their papers, on the basis of analytical dynamical considerations, they clarified the problem of the choice of the proper coordinates for expressing the quantum conditions. Sommerfeld here outlined their discussions on the relation between the quantum condition and the Hamilton-Jacobi general dynamics and stressed that his generalized quantum condition could be formulated whenever a coordinate system was found in which the Hamilton-Jacobi differential equation was of a separable variables type.

In Part II, as in the January 1916 paper, Sommerfeld, in order to verify his theory developed in Part I, considers the fine structure of hydrogen and hydrogen-type spectra on the basis of a relativistic quantum theory.⁹³ He discusses relativistic Keplerian motion in greater detail. Explicitly introducing a constant $\alpha = 2\pi e^2/hc$, the so-called fine structure constant, which determines the degree of deformation of elliptical orbits, he obtains an approximate relativistic expression of the energy of orbital motion as

$$W = -\frac{Nh}{(n + n')^2} \left(\frac{E}{e}\right)^2 \left[1 + \frac{\alpha^2}{(n + n')^2} \left(\frac{E}{e}\right)^2 \left\{ \frac{1}{4} + \frac{n'}{n} \right\} \right],$$

where N is the Rydberg constant, E the charge of the nucleus, and n and n' are azimuthal and radial quantum numbers. On the basis of this equation he discusses the general character of the fine structure of the spectral lines and carries out calculations concerning the number of the components of the fine structure, the distance between the components, and their intensity for a number of individual cases. From the results of his calculations he attempts to explain the observational results for hydrogen, ionized and neutral helium, and lithium.

In Part III Sommerfeld deals with the fine structure of the characteristic X-ray spectra.⁹⁴ From the experimental results obtained by Moseley and others,

⁹¹ P. S. Epstein, *op. cit.* (note 83).

⁹² K. Schwarzschild, *op. cit.* (note 83).

⁹³ A. Sommerfeld, *op. cit.* (note 1), II, 44-94.

⁹⁴ A. Sommerfeld, *op. cit.* (note 1), III, 125-167. See S. Nisio, "X-rays and Atomic Structure at the Early Stage of the Old Quantum Theory", *Jap. Stud. Hist. Sci.*, No. 8 (1969), 55-75.

Sommerfeld concludes that the frequency of the K_α -line is expressed by the equation

$$\nu_{K_\alpha} = K - L,$$

where $K = N((Z - k)^2/a^2)$, $L = N((Z - l)^2/b^2)$, a and b being 1 and 2 respectively, and where $(Z - k)$ and $(Z - l)$ are the effective charges on the nucleus. This equation indicates, he thought, that the K and L terms would behave similarly to those for hydrogen if the nuclear charge were replaced by the effective charge, and the electrons, at least in the innermost two rings, would describe hydrogen-like orbits. He could therefore apply the relativistic expression for the energy of orbital motion obtained in Part II to the motions corresponding to the K - and L -terms. The separations of the L -doublets thus obtained agrees satisfactorily with measurements for elements from $Z = 24$ (Cr) to $Z = 92$ (U), if l expressing the screening effect is made equal to 3.5 and k equal to 1.6. This result strengthened his conviction that the K and L orbits should be hydrogenlike.

Concluding his consideration, Sommerfeld pictures the atom model as follows: The difference between the optical spectra and the characteristic X-ray spectra is that, while the former is produced by the displacement of electrons outside the electron cloud, the latter is produced by the displacement of electrons belonging to the innermost part inside the electron cloud. The similarity of the spectral terms for the K -line and the hydrogen-type terms suggests that the electrons describing K and L orbits are hardly disturbed by the electron cloud. This explains the fact that the larger the quantum number in the case of optical spectra, and the smaller in the case of X-ray spectra, that is to say, the more distant the electrons taking part in the emission of radiation are from the electron cloud, the more the spectrum formula approaches a hydrogen-type formula.

Compared with the previous two papers, the revisions and additions in the *Annalen* paper may be summarized as follows: 1) The generalized quantum condition for a system of f degrees of freedom,

$$\iint dp_i dq_i = h \quad (i = 1, 2, 3, \dots, f)$$

is for the first time postulated explicitly as a fundamental hypothesis. In the previous paper he did use a similar condition, but it was not formulated with full generality. The region of the phase integrals is modified so as to give better agreement with experiment. 2) The quantization of the latitude variable is added. 3) The choice of the system of coordinates to be used is considered in connection with the theories of the Stark effect by Schwarzschild and by Epstein. 4) The relativistic Keplerian motion of electrons is discussed in greater detail and the importance of the fine structure constant as a spectroscopic constant is stressed. The available experimental data to be compared with the results of calculation, especially for X-ray spectra, had increased since Sommerfeld wrote the former papers.

It is very remarkable that Sommerfeld, throughout his 1916 article, is very much concerned to compare his calculations with experimental observations. His deep interest in such comparisons with observational results may be explained, I believe, by the fact that his quantum condition then had no sound theoretical foundation. As he stresses in Part I of his article, his quantum condition is merely a formal generalization of the quantum condition obtained for the hydrogen Balmer series.⁹⁵ Therefore good agreement of his calculations with observations must have been the only evidence that he could adduce in support of his generalized quantum theory.

VII. Methods of Quantization in 1911 and 1915-16

Comparing Sommerfeld's theories of 1911 and of 1916, one of the differences is the fundamental hypothesis for quantizing the system considered. Sommerfeld in 1916 postulates the relation,

$$\int dp dq = h, \quad (\text{a})$$

whereas in 1911 he used the condition,

$$\text{energy} \times \text{time} \sim h, \quad (\text{b})$$

or

$$\int (T - V) dt = h/2\pi. \quad (\text{b}')$$

The idea of quantizing a physical system by means of a condition of the form (a) is also found in Planck's paper of 1911 presented to the first Solvay Congress.⁹⁶ In that paper, as was noted in section II, Planck proposed to divide the phase plane of an oscillator into cells, for each of which

$$\int dp dq = h,$$

and to regard them as elementary domain of equal probability. In his December 1915 paper and 1916 *Annalen* paper, Sommerfeld, referring to "Planck's famous ellipses of a linear oscillator", repeats the same idea of dividing the phase space into elementary domains of equal probability.⁹⁷ In the latter paper, citing Planck's Solvay Congress paper, he explicitly writes that he considers "as the elementary domain of probability, according to Planck, a phase integral with finite extension",⁹⁸ $\int dp dq = h$.

As was mentioned in section II, Planck had already in his *Wärmetheorie* of 1906 and *Acht Vorlesungen* at Columbia University in 1909 derived the quantized

⁹⁵ A. Sommerfeld, *op. cit.* (note 1), I, 6.

⁹⁶ M. Planck, *op. cit.* (note 8), 99.

⁹⁷ A. Sommerfeld, *op. cit.* (note 1), I, 5 (note 80), 427.

⁹⁸ A. Sommerfeld, *op. cit.* (note 1), I, 5.

energy of resonator $U = nh\nu$ by dividing its phase plane into elementary domains of area h . This method of quantization was adopted in 1911 by F. Hasenöhr in his attempt to derive the Balmer formula.⁹⁹ At the 83th meeting of German Scientists and Physicians, he proposed to derive the Balmer formula by assuming a special kind of oscillator, for which he put Planck's quantum condition above in the form,

$$\int_0^{E_1} \tau dE = \int_{E_1}^{E_2} \tau dE \dots = \int_{E_m}^{E_{m+1}} \tau dE = h,$$

where E was the energy of the oscillator and τ was its period. Sommerfeld also noticed, in his paper presented to the same meeting, that Planck's radiation formula could be derived by considering elementary domains of equal probability without introducing the energy quantum.¹⁰⁰ There he cites Planck's Columbia lectures.¹⁰¹ These facts show that, by 1911, Planck's idea of quantizing the phase space had become well-known and had occasionally been used by others.

However, in dealing with aperiodic processes, which were the main target of Sommerfeld's effort in 1911, he did not use the method of dividing the phase space. This, it seems, was not without reason. For before 1911 the quantum concept had been used exclusively in problems involving statistical mechanical considerations such as black-body radiation and the specific heat of solids. The resonator and the atom that lie at the basis of these phenomena could be assumed to be periodic systems. On the other hand, the problems with which Sommerfeld was faced in 1911 were concerned with *elementary* and *aperiodic* processes. It would be quite natural that he should feel it necessary to coin a new type of quantum hypothesis. This appears to be the reason why Sommerfeld in 1911 used, instead of quantization of the phase space, a quantum hypothesis of type (b). In fact, he emphasizes the novelty of his subject by titling his paper presented to the Solvay Congress as "Application de la théorie de l'élément d'action aux phénomènes moléculaires non périodiques".

In contrast to this, in 1915-16 Sommerfeld based his theory on the quantization of the phase space. This is not an astonishing change, since sometime between 1913 and 1915 he turned his attention from aperiodic processes to periodic ones. After 1913 when he tried to elaborate the theory of photoelectric effect, he never dealt with the theory of aperiodic elementary process such as photoelectric effect, ionization, and bremsstrahlung. The system he considers in 1915-16 is the Bohr atom, which is naturally a periodic system. This change of subject should inevitably have entailed an alteration of the quantum condition used. In fact Sommerfeld himself, in the December 1915 paper, explains the

⁹⁹ F. Hasenöhr, "Über die Grundlagen der mechanischen Theorie der Wärme", *Phys. ZS.*, **12** (1911), 931-935.

¹⁰⁰ A. Sommerfeld, *op. cit.* (note 4), 1059 and 1067.

¹⁰¹ M. Planck, *op. cit.* (note 22).

reason for the alteration of the quantum condition. In the last section of that paper,¹⁰² having shown that for a periodic system the quantum condition $\int p dq = nh$ leads to a relation between the total energy of the system and the quantum of action h which resembles the quantum condition, energy \times time = h , used in the January 1911 paper on γ - and X-rays, he remarks that in spite of this resemblance there is a great difference in the interpretation of the quantum conditions. He emphasizes that the condition proposed here is exclusively for periodic systems whereas that of the 1911 paper was used for discussing aperiodic processes. In the same December 1915 paper, he stresses, in connection with the Stark effect, that he finds difficulty in applying his quantum condition to an aperiodic electron orbit.¹⁰³ It is now evident that Sommerfeld sharply distinguished periodic systems from aperiodic ones and assigned different types of quantum conditions to each of them.

Thus it may be concluded that Sommerfeld's 1916 theory was by no means the result of endogenous development of his 1911 theory. There was a discontinuity between them, which was brought about by his committing himself to the Bohr theory. The intervention of the Bohr theory was, it may therefore be said, indispensable for Sommerfeld to achieve his results. It should not be overlooked, however, that there was also a contribution to Sommerfeld's 1916 theory on the part of Planck.¹⁰⁴ The next section is devoted to a consideration of this.

VIII. Sommerfeld and Planck, 1915-16

Almost simultaneously with Sommerfeld, Planck published a generalized quantum theory for a system with many degrees of freedom. Planck's theory was developed in three successive papers which were presented on November 5 and December 3, 1915 to the German Physical Society¹⁰⁵ and on December 16

¹⁰² A. Sommerfeld, *op. cit.* (note 80), 454-458.

¹⁰³ A. Sommerfeld, *op. cit.* (note 80), 426 and 451.

¹⁰⁴ In this connection it may be noted that in the spring of 1915, a few months before Sommerfeld's presentation of the generalized quantum theory, W. Wilson and J. Ishiwara had proposed quantum conditions similar to those of Sommerfeld, independently of each other: W. Wilson, "The Quantum-Theory of Radiation and Line Spectra", *Phil. Mag.*, 29 (May, 1915), 795-802, dated March 1915. J. Ishiwara, "Die universelle Bedeutung des Wirkungsquantums", *Tokyo Sugaku Buturigakkai Kizi*, 8 (1915), 106-116, dated 4 April 1915. However, there seem to be no contributions to the formation of Sommerfeld's theory from these sources. In the first two papers of 6 Dec. 1915 and 8 Jan. 1916 Sommerfeld did not cite their papers. In the *Annalen* paper he first cited Wilson's paper but wrote, hearing von befreundeter Seite that A. Wilson had proposed the same quantum conditions as his own, he had read Wilson's paper but had not seen Ishiwara's: A. Sommerfeld, *op. cit.* (note 1), 9-10.

¹⁰⁵ M. Planck, "Die Quantenhypothese für Molekeln mit mehreren Freiheitsgraden, I, II", *Verh. d. Deutsch. Phys. Ges.*, 17 (1915), 407-418, 438-451; *Phys. Abh.*, 2, 349-360, 362-375.

to the Berlin Academy of Science.¹⁰⁶ The results obtained in these papers were eventually collected together, with revision, in a comprehensive paper which was received on April 13, 1916 and published in *Annalen der Physik*.¹⁰⁷ The quantum condition used by Planck is derived from the idea that the phase space could be divided into elementary cells for which

$$\int \prod_{i=1}^f (dp_i dq_i) = h^f.$$

The last condition will automatically be satisfied if Sommerfeld's condition is valid, though the reverse is not necessarily true. The parallels in the dates of the papers and in the quantum conditions of Planck's and Sommerfeld's theories lead us to suspect their mutual influence. But before rushing into a conclusion it is necessary to examine briefly the intention and the method of Planck's theory.

The motive behind Planck's theory was the question posed by Poincaré at the first Solvay Congress, that is, the question in what way is the phase space of a system with many degrees of freedom f divided by h .¹⁰⁸ In order to answer this question Planck considered the structure of the phase space. In the first paper, presented on November 5, 1915, he discusses the case in which the equi-energy surfaces divide the phase space into elementary cells whose extension $\int \prod_{i=1}^f (dp_i dq_i)$ is proportional to h^f . The second paper, presented on December 3, 1915, deals with the general case in which more than one family of surfaces determines the elementary cells. Among the individual cases discussed in this paper are the motion of a mass point under a central force, and the three-dimensional harmonic oscillator.

The third paper, presented on December 16, 1915, deals with the Keplerian motion of an electron with mass μ and electric charge $-\epsilon$ which moves around a fixed nucleus of charge $+E$. For the quantized elliptical orbit of this electron, Planck obtains the major axis a and ordinate of focus p as follows:

$$a = \frac{q'^2 h^2}{4\pi^2 \mu E \epsilon}, \quad p = \frac{q^2 h^2}{4\pi^2 \mu E \epsilon},$$

where q and q' are positive integers and $q \leq q'$. Now, in order to explain the spectral lines, he assumes that the frequency ν of the line is equal to the frequency of rotation of the electron which emits it. This frequency is given by

$$\nu = \frac{1}{q'^3} \frac{4\pi^2 E^2 \epsilon^2 \mu}{h^3},$$

¹⁰⁶ M. Planck, "Bemerkung über die Emission von Spektrallinien", *Sitzb. Preuss. Akad. Wiss.* (1915), 909-913; *Phys. Abh.*, 2, 376-380.

¹⁰⁷ M. Planck, "Die physikalische Struktur des Phasenraumes", *Ann. d. Phys.*, 50 (1916), 385-418; *Phys. Abh.*, 2, 386-419.

¹⁰⁸ M. Planck, *op. cit.* (note 8), 116-117.

which, in the case of the hydrogen atom, he argues, agrees with the frequencies of two lines in the violet region if $q' = 1$ and 2, and with those of lines in the red region if $q' = 3, 4, \dots$. For the radial component of the motion of the electron, Planck assumes the relation

$$h\nu = \left(\frac{1}{2} m\dot{r}^2 \right)_{\max}$$

between the maximum kinetic energy and the frequency of the spectral line originating from the elliptical motion, and from this he derives a formula expressing ν in terms of quantum numbers q and q' :

$$\nu = \frac{2\pi^2\mu E^2 e^2}{h^3} \left(\frac{1}{q^2} - \frac{1}{q'^2} \right),$$

which is identical with Bohr's formula for the Balmer series. He seems to have been satisfied himself that the Balmer formula could be obtained without recourse to the Bohr theory.

The cursory examination above is sufficient to show that both the intention and the method of Planck's theory is considerably different from those of Sommerfeld's. Whereas Sommerfeld developed his approach by generalizing the Bohr theory of hydrogen spectrum and, to substantiate it, discussed number of individual problems, Planck started from a general problem concerning the structure of the phase space. It may therefore be said that their theories emerged independently of each other from different ideas.¹⁰⁹ But once they had become acquainted with each other's work, they immediately recognized the intimate relation between their theories and started correspondence. Since Sommerfeld reported on Planck's theory on January 25, 1916 at the Munich colloquium,¹¹⁰ he must have examined Planck's first three papers before this date, and immediately after this colloquium the correspondence was begun. There remain five letters of Planck to Sommerfeld written in 1916, dated January 30, February 11, April 4, May 17, and November 17.¹¹¹ Though there seem to be no letters extant of Sommerfeld to Planck, Planck's letter of January 30 suggests that the correspondence was started by a letter from Sommerfeld written on January 27, 1916. Sommerfeld seems to have subsequently written to Planck on February 7, May 13, and in November, 1916. Their correspondence contributed much to the improvement of Sommerfeld's theory.

¹⁰⁹ Sommerfeld mentioned Planck's 1915 papers for the first time in his appendix, dated February 10, 1916, to the January 1916 paper (*op. cit.* note 84), and Planck mentioned Sommerfeld's papers in his *Annalen* paper (*op. cit.* note 107).

¹¹⁰ Register volume for Münchener physikalisches Mittwochs-Colloquium, *op. cit.* (note 65).

¹¹¹ These letters are kept deposited at the Institute of the History of Science, University of Stuttgart. The author thanks Prof. Hirosige for drawing her attention to these letters. She is also very grateful to Prof. A. Hermann who was kind enough to have the letters copied and sent to her.

Judging from Planck's letter of January 30, Sommerfeld's first letter was concerned with the problems of a three-dimensional harmonic oscillator, Keplerian motion, and spectra. In the letter of January 30, Planck praises Sommerfeld's work, and discusses problems concerning the variable in the phase space used in Planck's December 15 paper and the difference between the results of their calculations for the hermonic oscillator. In his second letter to Sommerfeld, dated February 11, Planck answers Sommerfeld's question about the limits of integration in the treatment of relativistic Keplerian motion. As mentioned in section VI, on February 10, 1916, Sommerfeld added to his January 1916 paper an appendix in which he alters the limits of integration for the azimuthal angle ϕ . Whereas in the text of the January 1916 paper he performed the integration of ϕ from 0 to the next perihelion, he now performs the integration from $\phi = 0$ to 2π . Saying that whereas the former calculation leads to disagreement with the observational results, the new limits of integration give a satisfactory agreement, he tries to justify this alteration by introducing a new coordinate system which rotates with the same velocity as the perihelion. Not being fully satisfied with this justification, Sommerfeld, on February 7, three days before he wrote the appendix in question, wrote to Planck asking the reason for the new limits of integration. Planck replies, in his letter of February 11, that the variable ϕ in his theory is a coordinate in the phase space which is independent of the shape of the electron orbit in three-dimensional physical space, and that if the integral for ϕ is taken from 0 to the value corresponding to the next perihelion, the region between 2π and that angle would be included twice.

Accepting Planck's explanation, Sommerfeld, in his *Annalen* paper, argues for the limits of integration $\phi = 0$ and 2π on the grounds of statistical mechanical considerations.¹¹² Take an ensemble of many similar systems describing Keplerian elliptical orbits of the same shape. The systems in this ensemble differ from each other in the direction of their perihelion. To take the integral for ϕ in the phase space from 0 to 2π , therefore, is equivalent to taking an average over the ensemble. This procedure need not be changed even when one considers relativistic Keplerian motion.

As for the harmonic oscillator, there was a discrepancy between Planck and Sommerfeld. In the paper presented on December 3, 1915, Planck derives, for the quantized elliptical orbits of the mass point m , a relation between the semi-major axis a and the semi-minor axis b given by

$$(a - b)^2 = \frac{2nh}{\pi m\omega},$$

where ω is the frequency of the oscillator.¹¹³ On the other hand Sommerfeld

¹¹² A. Sommerfeld, *op. cit.* (note 1), I, 7-8.

¹¹³ M. Planck, *op. cit.* (note 105), II, 448; *Phys. Abh.*, 2, 372.

writes in his letter of January 27, that his theory gives the relation,

$$(a - b)^2 = \frac{nh}{\pi m \omega}.$$

Having discussed the discrepancy with Planck in their letters of January 27 and 30, Sommerfeld modified his original limits of integration in the same way as in the case of relativistic Keplerian motion and obtained the same result as Planck. He discusses this modification in detail in the section entitled "Vergleich mit der Planckschen Theorie" of Part I of his *Annalen* paper, where he acknowledges that this alteration is due to "Planck's kind remarks in his letter".¹¹⁴

In Part II of the *Annalen* paper, Sommerfeld discusses the relativistic Keplerian motion in fuller detail than in the previous, January 1916 paper.¹¹⁵ This seems to have been prompted by Planck's suggestions. Forwarding his paper to the *Annalen*, Planck wrote, on April 4, to Sommerfeld, noting that since it had been shown that in the case of quasi-periodic relativistic Keplerian motion there existed a finite lower limit of the angular momentum, $p_0 = eE/c$, it seemed natural to put the quantum condition for the angular momentum,

$$p - \frac{eE}{c} = \frac{nh}{2\pi},$$

instead of Sommerfeld's $p = nh/2\pi$, where p_0 was assumed to be equal to 0, and he asked Sommerfeld to examine what modifications would be produced in his calculation of spectra using Planck's quantum condition. Replying on May 13 to Planck, Sommerfeld seems, judging from Planck's letter of May 17, to have stated that his calculations had proved that Planck's quantum condition did not lead to the Balmer formula and he would like to insist on his own quantum condition $p = nh/2\pi$. The result of this discussion with Planck is incorporated into his *Annalen* paper as an independent section "Prüfung eines von Planck befürworteten Quantenansatzes an der Erfahrung. Prüfung der Relativitätskorrektion für Kreisbahnen".¹¹⁶ There Sommerfeld states that whereas it seems natural to begin the division of phase plane from p_0 , not from 0, Planck's condition leads to results at variance with experimental observations.¹¹⁷

Sommerfeld was naturally aware of the difference between his theory and Planck's. In the appendix to the January 1916 paper and in a few places in the *Annalen* paper, he emphasizes the difference between their theories. In the appendix to the January paper, Sommerfeld points out that, although in the case of Keplerian motion under the influence of the Coulomb force his quantum condition coincides with Planck's, his explanation of the Balmer series, being based

¹¹⁴ A. Sommerfeld, *op. cit.* (note 1), I, 33-36.

¹¹⁵ A. Sommerfeld, *op. cit.* (note 1), II, 47-52.

¹¹⁶ A. Sommerfeld, *op. cit.* (note 1), II, 57-62.

¹¹⁷ A. Sommerfeld, *op. cit.* (note 1), II, 61.

on the concept of quantum transition, fundamentally differs from Planck's.¹¹⁸ In the opening paragraph of Part I, section I of the *Annalen* paper, he compares his general quantum condition, $\int dq_i dp_i = h, i = 1, 2, \dots, f$, to that proposed by Planck. He says that for a system with f degrees of freedom, whereas Planck, on the basis of general considerations, divided one quantum condition, $\int \prod_{i=1}^f (dq_i dp_i) = h^f$, in this case, into f quantum conditions taking into account the nature of the system and the coherence of its degrees of freedom, he, on the basis of semi-empirical consideration of the Balmer series, expresses the quantum condition for each of the f degrees of freedom of the system as

$$\int dq_i dp_i = h, \quad i = 1, 2, \dots, f.^{119}$$

In the section "Vergleich mit der Planckschen Theorie . . .", comparing the results obtained by his theory for a three-dimensional oscillator with the results obtained by Planck's, Sommerfeld emphasizes that though both results agree in the main, with each other, their explanations of the frequency of spectral lines are based on entirely different principles.¹²⁰

These repeated references to Planck's theory of 1915-16, however, suggest how deep was the impression made on him by Planck's theory. In fact, our preceding examination reveals that various additions and alterations in Sommerfeld's *Annalen* paper resulted from his consideration of Planck's theory of 1915-16.

Finally it may also be noted that Sommerfeld in turn also influenced Planck. Concerning the physical interpretation of the quantized state, Planck's position had remained the same since 1911, when he developed a new version of the theory of radiation on the assumption that the resonator could assume continuous values of energy but would emit radiation only when it reached definite discrete values of energy. In accordance with this conception he had supposed that the representative point in the phase space of a physical system could occupy any point in the elementary cells of finite extension h^f . The points on the boundary of these cells are, in his opinion, distinguished from the interior points of the cells only by the peculiarity that only in the states represented by these points could the system emit radiation.¹²¹ This view is quite opposed to that of Sommerfeld, who believed that the electron in an atom could move only in quantized orbits, and explicitly rejected Planck's view.¹²² On the other hand, Planck was

¹¹⁸ A. Sommerfeld, *op. cit.* (note 86), 498-499.

¹¹⁹ A. Sommerfeld, *op. cit.* (note 1), I, 5-7.

¹²⁰ A. Sommerfeld, *op. cit.* (note 1), I, 33-34.

¹²¹ M. Planck, *op. cit.* (note 106), esp. 912.

¹²² A. Sommerfeld, *op. cit.* (note 1), I, esp. 12-13.

greatly impressed by Sommerfeld's relativistic theory of the fine structure of the spectral line.¹²³ He talked about it in detail on April 4, 1916 at a colloquium of his institute.¹²⁴ It is most likely because of this impact that in his *Annalen* paper Planck feels it necessary to allude the "fundamental problem" of whether or not the representative point could fill the elementary cells of the phase space. In this paper Planck restricts his consideration to the structure of the phase space and, at the end of the paper, remarks that it is impossible to apply his general result to thermodynamic problems of ideal gases without setting up a special assumption concerning this fundamental problem.¹²⁵ In a paper dealing with the entropy of atomic systems, published in 1916,¹²⁶ he bases his calculations on his original view that the representative point could fill the interior of the elementary cells on the grounds that this view seems preferable to the contrary one. Subsequently, in 1917, Planck developed a theory of rotational spectra based on his view with the express purpose of deciding by comparison with experiment which view of the quantization of the phase space was to be adopted.¹²⁷

That Planck turned his attention to spectra shows a remarkable change of attitude on his part, though he seemed still to insist on his own view, because he had so far paid little attention to spectra. In January 30, 1916 he himself wrote to Sommerfeld that his consideration of spectral emission in his 16 December 1915 paper was only a digression in a region which he had seldom entered.

IX. Conclusion

Sommerfeld's achievement in 1916 was immediately recognized and appreciated by contemporary physicists, who admired his mathematical skill. H. A. Lorentz, the mentor of physicists of the time, wrote to Sommerfeld on February 14, 1917: "You have reached one of the most beautiful results in theoretical physics."¹²⁸ Bohr, too, welcomed the Sommerfeld theory with enthusiasm, and on receiving Sommerfeld's two papers of December 1915 and January 1916 wrote to the latter: "I do not think that I have ever enjoyed the reading of anything more than I enjoyed the study of them, and I need not say that not only I but

¹²³ Planck's letter to Sommerfeld, dated February 11, 1916. Sommerfeld's theory of the fine structure of the spectral line seemed to confirm the validity of the relativistic formula for the electron mass. See T. Hirose, *op. cit.* (note 24), 36 and W. Pauli, *Relativitätstheorie*, (Leipzig/Berlin, 1921) 636-637. This would probably also have contributed to enhancing Planck's opinion of Sommerfeld's theory.

¹²⁴ Planck's letter to Sommerfeld, dated May 17, 1916.

¹²⁵ M. Planck, *op. cit.* (note 107), 418.

¹²⁶ M. Planck, "Über die absolute Entropie einatomiger Körper", *Sitzb. Preuss. Akad. Wiss.* (1916), 653-667; *Phys. Abh.*, 2, 420-434.

¹²⁷ M. Planck, "Zur Theorie des Rotationsspektrums. Vorl. Mitt.", *Verh. d. Deutsch. Phys. Ges.*, 17 (1917), 43-47; *Phys. Abh.*, 2, 453-457. M. Planck, "Zur Theorie des Rotationsspektrums. I, II", *Ann. d. Phys.* 52 (1917), 491-505, 53 (1917), 241-256; *Phys. Abh.*, 2, 458-472, 473-488.

¹²⁸ Lorentz' letter to Sommerfeld, Haarlem, February 14, 1917.

everybody here has taken the greatest interest in your important and beautiful results. . . ."¹²⁹ Sommerfeld's theory was also important in connection with the theory of relativity, because its success in explaining the fine structure of spectral lines was thought to provide a confirmation of the relativistic formula for the mass of the electron. As was mentioned in the previous section, Planck was greatly impressed by this result.

Sommerfeld continued publishing papers on the quantum theory of line spectra and the structure of the atom.¹³⁰ In 1918 he attempted to determine the arrangement of electrons in the inner part of the atom on the basis of X-ray spectra.¹³¹ All these achievements are incorporated into his celebrated book *Atombau und Spektrallinien*,¹³² which was written on the basis of his lectures during the 1917/18 winter semester and published in 1919. In the next year, without having enough time to make essential revisions,¹³³ he published the 2nd edition. This book became the standard text-book of theoretical spectroscopy and went through six editions, each edition reflecting the development of this branch of physics from 1916 through 1946.

The foregoing analysis of Sommerfeld's achievement in quantum theory seems to reveal the characteristic style of his work in theoretical physics. His contribution to quantum theory essentially consists not in introducing new fundamental ideas, but in mathematically elaborating theoretical attempts, whether they are well founded or problematic. It has been said that in general his skill was fully demonstrated in the mathematical manipulation of problems in various fields of physics and technology.¹³⁴ It may be said that, in the final analysis, he was interested in quantitative agreement between experiments and his detailed calculations. His approach to the problems of physics makes a sharp contrast with that of Einstein, who was interested primarily in the fundamental principles in physics and in the formal consistency of theory. To take an example, in

¹²⁹ Léon Rosenfeld and Erik Rüdinger, "The Decisive Years, 1911-1918" in S. Rozental (ed.), *Niels Bohr* (Amsterdam/New York, 1967), 67.

¹³⁰ For example, A. Sommerfeld, "Zur Quantentheorie der Spektrallinien. Ergänzungen und Erweiterungen", *Sitzb. Bayer. Akad. Wiss.* (1916), 131-182; A. Sommerfeld, *op. cit.* (note 84); A. Sommerfeld, "Zur Quantentheorie der Spektrallinien. Intensitätsfragen", *Sitzb. Bayer. Akad. Wiss.* (1917), 83-109.

¹³¹ A. Sommerfeld, "Atombau und Röntgenspektren, Teil 1", *Phys. ZS.*, **19** (1918), 297-307. For Sommerfeld's work on X-rays and atomic structure, see S. Nisio, *op. cit.* (note 94).

¹³² A. Sommerfeld, *Atombau und Spektrallinien*, (Braunschweig, 1919).

¹³³ A. Sommerfeld, *Ibid.*, 2nd ed., p. viii.

¹³⁴ See M. Born, "Sommerfeld als Begründer einer Schule", *Naturwiss.*, **16** (1928), 1035-1036; W. Pauli, "Sommerfelds Beiträge zur Quantentheorie", *Naturwiss.*, **35** (1948), 129-132; P. P. Ewald, "Prof. A. Sommerfeld", *Nature*, **168** (1951), 364-366; W. Heisenberg, "Arnold Sommerfeld", *Naturwiss.*, **38** (1951), 337-338; Max von Laue, "Sommerfelds Lebenswerk" *Naturwiss.*, **38** (1951), 513-518; In F. Bopp and H. Kleinpoppen (eds.), *Physics of the One- and Two-Electron Atoms* (Amsterdam, 1969), F. A. Bopp, P. P. Ewald, A. Hermann, B. L. Van der Waerden, H. Welker, and W. Heisenberg gave historical consideration of Sommerfeld's work.

discussing relativistic Keplerian motion, Sommerfeld chose a quantum condition which led to a result agreeing with experimental observations. On the other hand, Einstein's comment, at the first Solvay Congress, on Sommerfeld's calculation of the X-ray energy that numerical agreement with observational data would by no means confirm by itself the validity of the theory shows a marked difference between their approaches to the problem. It may also be remembered that in his 1915–16 papers, Sommerfeld devoted many pages to comparison with experiments. In the introduction to *Atommechanik* (1924) M. Born, comparing his book with Sommerfeld's *Atombau*, suggested that Sommerfeld described experimental results in too much detail.¹³⁵

Sommerfeld was aware of this tendency and accepted his way of doing theoretical physics as a necessary one. In his correspondence with Stark and Planck (See Section II and VIII) Sommerfeld's confidence in his own mathematical ability is clearly seen. In a letter to Einstein on January 11, 1922, Sommerfeld writes "I can only develop quantum techniques, whereas you must make your philosophy."¹³⁶ He demanded that his students follow his way and made them practise solving individual little problems.¹³⁷

It is interesting to consider his worldview or view of physics on the basis of which he could have such confidence in his way of doing theoretical physics. What view of physics could justify his method of research? What goals did he have in physical research? It is to be regretted that Sommerfeld, as can easily be imagined, published very little on philosophical problems. One of the few is a lecture on "Wege zur Physikalischen Erkenntnis" delivered on May 1, 1933 at the Royal Society of Edinburgh.¹³⁸ In this lecture Sommerfeld reviewed Planck's book¹³⁹ of the same title, which is a collection of lectures and had just been published, and, endorsing Planck's opinion as regards Mach's positivism, stated that laws of nature really existed independently of human sense. As regards abstract mathematization of modern physics, as in the case of quantum mechanics, he declared his belief that mathematics plays an important role in physical cognition and said: "Nature is indifferent to our mathematical ability. Nature is a much better mathematician than we are. Laws of nature are formulated not by the help of elementary mathematical methods, but by the highest and most effective ones."

Sommerfeld advanced similar view in his ceremonial address at the Munich

¹³⁵ M. Born, *op. cit.* (note 86), p. viii.

¹³⁶ A. Hermann (ed.), *op. cit.* (note 41), 97.

¹³⁷ Heisenberg says that Sommerfeld insisted that students of theoretical physics had to practise the calculation of little problems before discussing fundamental philosophical problems: W. Heisenberg, *Der Teil und das Ganze* (München, 1967), pp. 31–32.

¹³⁸ A. Sommerfeld, "Wege zur Physikalischen Erkenntnis", *Scientia* (1936), 181–187; *Gesammelte Schriften*, 4, 609–615.

¹³⁹ M. Planck, *Wege zur Physikalischen Erkenntnis* (Berlin, 1933).

Academy of Science in July 1925,¹⁴⁰ and spoke of a revival of the Pythagorean belief in integral numbers. Earlier than this, in the preface to his *Atombau* of 1919, he referred to “number mystics” saying: “What we are nowadays hearing of the language of spectra is a true music of the spheres within the atom, chords of integral relationships, an order and harmony with a manifold variety.”¹⁴¹

His amicable attitude towards “number mystics” is most likely linked with his position which attaches prime importance to the numerical agreement of theoretical calculation with observational data. Having such a view of nature and of theoretical physics, he must have regarded it as primarily important to attack problems which may be expected to allow us to draw numerical results which can be compared with experimental observations. It is therefore understandable that he was not much concerned with fundamental, philosophic problems in physics and showed his great mathematical skill in working out theoretical ideas presented by fellow physicists.

It may be said that Sommerfeld was typically a physicist in the age of professionalism in the sense that he utilized his expert competence to solve problems posed to him, regardless of their philosophical background or the field to which they belong. It is however to be emphasized that Sommerfeld, with such an inclination, was just the theoretical physicist who really met the demands of the 1910's for the development of quantum theory.

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¹⁴⁰ A. Sommerfeld, “Die Bedeutung der Röntgenstrahlen für die heutige Physik”, *Gesammelte Schriften*, 4, 564-579, esp. 573-574.

¹⁴¹ A. Sommerfeld, *op. cit.* (note 132), pp. vii-viii. For Sommerfeld's “number mystics” see P. Forman, “Weimar Culture, Causality, and Quantum Theory, 1918-1927; Adaptation by German Physicists and Mathematicians to a Hostile Intellectual Environment”, *Hist. Stud. Phys. Sci.*, 3 (1971), 1-115, esp. 49-50.