The Genesis of the Bohr Atom Model and Planck's Theory of Radiation

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1. Introduction

In 1964–65 we considered the origin of Bohr's quantum theory of atomic constitution and concluded as follows:[1]

1. In the first decade of the twentieth century, rather the chemical consideration afforded the useful foundation for constructing an atomic model than the spectrum. J. J. Thomson's positive sphere model, which was the most successful theory of atomic structure of the time, aimed at explaining the chemical properties of elements. It may also be stressed that Thomson's model was the first theory that formulated the two historically important notions in the development of atomic model, that is, the notions that the chemical properties of an atom were determined by the configuration of electrons in the atom, and that the intra-atomic electrons distributed themselves in several rings.
2. Bohr's consideration of atomic constitution was begun with the intention to give an explanation of the chemical properties of atoms and molecules. In this attempt Bohr inherited the aim as well as the two fundamental notions above of Thomson's theory.
3. The common understanding[2] that Bohr formulated his theory by borrowing the idea of quantizing the angular momenta of orbital electrons from J. W. Nicholson is quite contrary to the fact. Nicholson cannot be said to have put forth such a clear idea that the electron orbit would be determined by the condition of quantized angular momentum. Furthermore Bohr's condition for fixing the electron orbit was not stated in terms of the angular momentum.
4. Bohr's actual quantum condition $W=(\tau h/2)\omega$, where W is the amount of energy emitted during the binding of an electron by the nucleus, ω the number of rotation of the electron on its final orbit, and τ an integral

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[1] T. Hirosige and S. Nisio, "Formation of Bohr's Theory of Atomic Constitution", *Jap. Stud. Hist. Sci.*, No. 3, 6–28 (1964). T. Hirosige, "On the Background of Bohr's Theory of Atomic Constitution", *Actes du XI^e Congrès international d'Histoire des Sciences, III*, 430–434 (1965).

[2] For example, E. T. Whittaker, *A History of the Theories of Aether and Electricity, II. The Modern Theories*, Thomas Nelson, London, 1953, pp. 107–109.

number, is inferred to be introduced by an analogy of, or at a suggestion from, Planck's theory of radiation of 1911–12.

5. Planck's theory is remarkable in its foreshadowing some of the novel features of post-Bohr theory of radiation. First, it discarded the classical notion that the oscillation of charged particle directly produces the electromagnetic waves. Second, the emission of radiation is regarded as an instantaneous and statistical process. Lastly, the emission of radiation is suspected to be connected with a binding or a release of the electron.

Recently John L. Heilbron and Thomas S. Kuhn have published an interesting article on "The Genesis of the Bohr Atom,"[3] using plenty of materials gathered by the project "Sources for History of Quantum Physics". Making also use of Bohr's personal correspondences, they have attempted to reconstruct a possible course of the development of Bohr's thought from his doctoral thesis of 1911 to the composition of his renowned paper "On the Constitution of Atoms and Molecules" of 1913. In their paper they have criticized our assertion about the relation of Bohr's theory to Planck's theory. Challenged by their criticism, we have reconsidered the problem anew by examining also some new materials to which we formerly did not have access. The following is partly a reply to their criticism based on this reconsideration. It will also present some new remarks about the problem.

2. Conclusions of Heilbron and Kuhn

The major results of Heilbron and Kuhn's paper may first be summarized as follows:

1. Throughout all the efforts of Bohr from the year 1911 on, various germs of the ideas involved in the 1913 paper may be seen. Especially it is suggested that the notion of the relation $W \sim h\nu$, where W is the energy and ν is the frequency of the system considered, was gradually formed through his deep concern with J. J. Thomson's doublet model of atom[4], which was introduced in order to explain the photo-electric effect, and also with the investigations on the magneton pursued by number of distinguished physicists since 1911.
2. The influence of Nicholson's theory of spectra is stressed. Through a serious struggle with Nicholson's theory, Bohr became to turn his attention to the spectrum and reached such an important idea as the existence of excited states.
3. Bohr's quantum condition $W = (\tau h/2)\omega$ is concluded to have been obtained semi-empirically from Balmer's formula. Namely, Bohr should have fixed the factor 1/2 so as to give a numerical agreement with the Balmer formula.

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[3] John L. Heilbron and Thomas S. Kuhn, "The Genesis of the Bohr Atom", *Hist. Stud. Phys. Sci.* **1**, 211–290 (1969) (Hereafter cited as Heilbron-Kuhn)

[4] J. J. Thomson, "On the Theory of Radiation", *Phil. Mag.*, (6) **20**, 238–247 (1910).

4. Consequently, what Bohr called "relation to Planck's theory" is concluded to be merely an *ad hoc* rationalization to enhance the plausibility of his theory. Our former paper which attached much importance to Planck's theory should therefore be criticized.

Of these conclusions of Heilbron and Kuhn, the first is possibly right. We agree with them in admitting that the investigation on the electron theory of metal and interests in Thomson's doublet model and the magneton would have formed the background of Bohr's theory. But we are sceptical about suggesting that Bohr had formed a clear conception of the theory of atomic constitution since very early. In his thesis, having derived the Rayleigh-Jeans formula by a quite general consideration, Bohr asserts that "it therefore seems that it is impossible to explain the law of heat radiation which agrees with the experience if one retains the foundation on which the electromagnetic theory is based." And he continues: "This failure has very likely resulted from this, that the electromagnetic theory does not agree with the real conditions and can only give correct results when it is applied to a large number of electrons (as in ordinary bodies) or to determine the average motion of a single electron in a comparatively long time (such as in the calculation of the motion of cathode rays), but cannot be used to investigate the motion of a single electron in a short time".[5] It is quite clear that already at this time Bohr was decidedly convinced of inadequacy of the classical electromagnetic theory. At the same time, however, it is noteworthy that Bohr does not here mention the quantum explicitly. Though he cites papers of Einstein (1909) and Planck (1910), he keeps silence on the necessity of h throughout his thesis. This silence might indicate that at this moment Bohr was not absolutely sure about the inevitability of introducing h into the problem of atom.

Heilbron and Kuhn's assertion on the influence of Nicholson seems to be tenable. They however do not positively deny the myth that Bohr borrowed the quantum condition from Nicholson. In the paper cited by them Russell McCormmach reckons the quantization of angular momentum as one of "the various possible direct influences of Nicholson's work on Bohr's thought."[6] It is evident that Bohr was possibly influenced by Nicholson when he, in the latter part of his paper, introduced the quantum condition of angular momentum as the basis of considering the atomic and molecular systems, since he writes that "the possible importance of the angular momentum in the discussion of atomic systems in relation to Planck's theory is emphasized by Nicholson."[7] But we would rather like to emphasize again our former assertion that Bohr's original quantum condition was not borrowed

[5] N. Bohr, *Studier over metallernes elektrontheori*, Diss., Copenhagen, 1911, p. 103.

[6] R. McCormmach, "The Atomic Theory of John William Nicholson", *Arch. Hist. Exact Sci.*, **3**, 160–184 (1966), esp. p. 179.

[7] N. Bohr, "On the Constitution of Atoms and Molecules, Part I", *Phil. Mag.*, (6) **26**, 1–25 (1913), esp. p. 15.

from Nicholson.*

Problems related to Heilbron and Kuhn's conclusions 3 and 4 will be discussed hereafter.

Heilbron and Kuhn regard the Balmer formula as having afforded Bohr the clue for the final solution. As a consequence of this view, they interprete Bohr's reference to Planck only as general and formal one. The interpretation that Bohr's condition $W = (\tau h/2)\omega$ was devised so as to give the Balmer formula is shared also by L. Rosenfeld.[8] And Bohr himself, in his Rutherford lecture of 1961, states that the idea struck him "in the early spring of 1913 that a clue to the problem of atomic stability directly applicable to the Rutherford atom was offered by the remarkable simple laws governing the optical spectra of the elements."[9] But one must be cautious in hearing Bohr's statements. His recollection in later years is often inaccurate. To take an example, on which Heilbron and Kuhn too make a remark,[10] in the interview conducted by Kuhn for the project of Sources for History of Quantum Physics,[11] Bohr says that the central problem in the earliest study on the Rutherford atom model was the *electromagnetic* instability. But all the contemporary documents indicate that in those days Bohr was actually concerned with the *mechanical* stability.

In view of such a circumstance, it seems reasonable presumably to doubt Bohr's story of the Balmer formula. We should rather rely on Bohr's contemporary writings than his later recollection. We first note the composition of his 1913 paper. The part I of this paper bears the title "Binding of Electrons by Positive Nuclei" and the quantum condition is introduced in the course of discussing the binding process. Should this be regarded only as a matter of style of presentation? As has been documented by Rosenfeld,[12] there was a fight between Bohr and Ruthurford about the length of the paper. Bohr, impatient in exchanging letters, hurried to Manchester to fight out the issue and succeeded in defending all the composition of his paper against Rutherford's demand that the paper be written as shortly as possible. Bohr insisted to retain all the lengthy argumentation to its full detail. It is therefore natural to suppose that the discussion of the binding of electron was, for Bohr, not a trivial one but of essential significance.

* Recently one of us (Hirosige) had an opportunity to talk with McCormmach, when the latter definitely denied the myth about Bohr's quantum condition. He told that he regarded it as Nicholson's most important influence that his theory turned Bohr's attention to the new problem, i.e. the spectrum, which Bohr had theretofore disregarded.

[8] L. Rosenfeld, "Introduction" to N. Bohr, *On the Constitution of Atoms and Molecules*, Munksgaard, Copenhagen, 1963, p. XL. (Hereafter cited as Rosenfeld)

[9] N. Bohr, "Reminiscences of the Founder of Nuclear Science and of Some Developments based on his Work" in J. B. Birks (ed.), *Rutherford at Manchester*, Heywood, London, 1962, pp. 114–167, esp. p. 122.

[10] Heilbron-Kuhn, p. 282, note 160.

[11] Interviews II (The roman numeral refers to the number of the five interviews with Bohr conducted by the project "Sources for History of Quantum Physics").

[12] Rosenfeld, p. XLV. See also N. Bohr, Ref. 9, pp. 127–129.

3. Spectrum or Binding?

As was shown by Rosenfeld,[13] it was at the beginning of February 1913 that Bohr was first acquaninted with, or made to direct his attention to, the Balmer formula at a suggestion of his friend H. M. Hansen. However, even before that date Bohr had already thought that his theory of atomic constitution was quite near to its completion. In his number of letters written from the late 1912 to the beginning of February 1913, we see him state that he will soon be able to publish his theory. On November 4, 1912, Bohr writes to Rutherford that he hopes to finish the paper on the atom in few weeks.[14] And on January 31, 1913, he again writes a letter to Rutherford and expresses his expectation that he will very soon be able to send his paper on the atom.[15] Five days later, he writes to his Swedish friend C. W. Oseen that he hopes soon to publish the work on the atom. He also says that he is in a hurry because the problem seems to him very urgent.[16] Finally, two days later, in a letter to G. von Hevesy he writes that he has made some progress in the latest time, and hopes to be very soon able to publish his work on the structure of the atom.[17]

The problem with which Bohr was occupied during these days is most likely the binding of electron by a nucleus. In his Christmas card of 1912 to his brother Harald, he writes his "calculations would be valid for the final, chemical state of the atoms, whereas Nicholson would deal with the atoms sending out radiation, when the electrons are in the process of losing energy before they have occupied their final position."[18] This passage seems to us to suggest that Bohr was led to the problem of the binding of electron in struggling with Nicholson's theory. In his letter of January 31, 1913 to Rutherford we find a clearer indication that he was thinking of the binding process. In that letter he writes that the system considered in his calculation is the one by the formation of which the greatest possible energy has been radiated out, i.e. the atom in its permanent state. A week later, on 7th February, he clearly states in his letter to Hevesy that he assumes that the atom is formed by a successive binding of electrons and that the energy radiated out during this binding process is proportional to $h\omega$, where ω is the frequency of rotation of the electron in its final orbit.

After the letter above to Hevesy, but before the beginning of March 1913, Bohr was acquaninted with the Balmer formula and on March 6 he sent the first draft of his paper to Rutherford. In the letter accompanying it he writes that the first chapter of the paper deals with the problem of emission of line-spectra con-

13 Rosenfeld, pp. XXXIX–XL.

14 BSC 6.3 (BSC stands for the Bohr Scientific Correspondence in the Sources for History of Quantum Physics. The first arabic numeral refers to the film number and the second to the section number).

15 BSC 6. 3

16 BSC 5. 4

17 BSC 3. 3

18 Rosenfeld, p. XXXVI.

sidered from the viewpoint described in his "former letter."[19] This "former letter" cannot be but the letter dated January 31. For Rutherford's answer to Bohr's letter of January 31 is written on February 24, in which he describes recent progress of the experiments being carried out in his laboratory, and Bohr's letter above of March 6 begins with his gratitude for hearing about the works under way in Rutherford's laboratory. Thus the letter of March 6 is the first letter of Bohr that he wrote to Rutherford after January 31. Then, that Bohr writes he considers the problem of spectrum from the viewpoint described in the former letter should be interpreted to indicate that he has applied his theory in hand to the spectrum.

Together with the consideration in the previous section, this seems to suggest that by the consideration of the binding process Bohr, before the beginning of February 1913, had reached the formulation which he then thought as nearly the completion of his theory and that the progress during Feburary, though, retrospectively, of epoch making importance for the subsequent development of quantum physics, was only to extend that theory to the spectrum. In the letter of March 6, Bohr states that from his former point of view it seems possible to give a simple interpretation of the hydrogen spectrum and that the result of calculation is in close quantitative agreement with experiment. This passage of Bohr's letter is quite in accordance with our interpretation above. If the Balmer formula afforded him the decisive clue for the completion of his theory, he would never have written in such a manner. It seems most natural to read the passage in question as expressing his satisfaction that his theory is strengthened by the success of its application to the hydrogen spectrum. Thus the Balmer formula may be said to have served Bohr rather as a confirmation of the theory which he had already had in hand, than as the final clue for his theory.

Heilbron and Kuhn states that it was "simply to make his model produce the Balmer formula" that Bohr put the proportional constant K between the kinetic energy and the frequency of rotation of an electron equal to $h/2$.[20] According to their version, Bohr is supposed to have proceeded this way: the Balmer formula $\nu = 3.29025 \times 10^{15}(1/4 - 1/m^2)$ multiplied by h is compared with the expression of the binding energy of electron $W = \pi^2 me^4/2K^2$, which Bohr seems to have obtained in his Rutherford memorandum of June or July 1912. Putting $K = \tau h/2$, he would obtained $W = 2\pi^2 me^4/\tau^2 h^2$. Divided by h, and if numerical values are substituted for e, e/m, and h, this gives $3.1 \times 10^{15}/\tau^2$, which agrees with the running term $3.29025 \times 10^{15}/m^2$ of the Balmer formula within a difference of 7%. Heilbron and Kuhn suppose that though before this time Bohr had already estimated K at a value something near to $h/2$, yet he had not been able to make the final decision about the value of K.[21]* They assert that the Balmer formula was the very clue

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[19] BSC 6. 3

[20] Heilbron-Kuhn, p. 271, note 99.

[21] Heilbron-Kuhn, p. 262, note 130.

* When Hirosige recently discussed the problem of $K = h/2$ personally with Kuhn, the latter

which enables Bohr to decide upon the K. But if this was the case, Bohr would have to come upon both the idea of electron transition and the factor $K = h/2$ at the same time. Does not this seem somewhat acrobatic, if not impossible, however ingenious as Bohr may be?

When interviewed by Kuhn and others, answering the question how did Bohr obtain $K = h/2$, he insists that he obtained it "very very accurately" by a correspondence.[22] He also emphasizes that it was he who first found that there is something in the spectrum. But Bohr's late memories of earlier events are in general unreliable. Above statement too is betrayed by his own words in his lecture delivered at the Physical Society in Copenhagen on December 20 of that crucial year, 1913. In this lecture, after deriving the energy levels of hydrogen atom by a correspondence method, he clearly states that originally he was led to the result not by this method but by an analogy with Planck's oscillator.[23] According to Rosenfeld, Bohr told him more than once; "As soon as I saw Balmer's formula, the whole thing was immediately clear to me."[24] But "whole thing" of what? If we read this as stating that the whole problem of the atomic structure was solved by the Balmer formula, are we not prejudiced by the popular version of the history of quantum theory? There seems to be no reason that prevents us from interpreting this as saying that it was immediately clear how to explain the Balmer formula by the theory he had already constructed. One should carefully free himself from the prejudice that Bohr's theory is the theory of atomic spectra. As was stressed at the beginning of the present paper, Bohr's theory was originally intended to explain chemical properties of the atom.

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told that Heilbron and Kuhn too, at first, had suspected that Bohr would have had a theoretical ground for $h/2$ (See also the note 109 on page 255 of their article). But after they found the footnote which Bohr added afterward in his letter to Hevesy of February 7, 1913, explained Kuhn to Hirosige, they became doubtful about this and finally reached the conclusion that Bohr estimated the value $h/2$ by a numerical consideration. In the footnote in question, Bohr says that the proportional constant between the energy radiated out and the frequency of rotation is not the Planck constant but his constant multiplied by a numerical factor. He adds that this is to be expected from theoretical considerations. Referring to this footnote Heilbron and Kuhn, in their paper, asserts that had Bohr made the final decision $K = h/2$ at this time, he would presumably have specified 1/2 rather than "a numerical factor" (p. 262, note 130). We, however, note that Bohr also writes that it is expected "from theoretical considerations". Anyway how to interprete this Bohr's additional passage is very delicate. But this passage does not seem to us necessarily to prevent us from supposing that Bohr had had a theoretical estimate of K.

[22] Interviews I.

[23] N. Bohr, "Om brintspektret", *Fysisk Tidsskrift* **12**, 97 (1914); "On the Spectrum of Hydrogen" in N. Bohr, *The Theory of Atomic Spectra and Atomic Constitution*, Cambridge Univ. Press, London, 1922, pp. 1–19, esp. p. 14.

[24] Rosenfeld, p. XXXIX.

4. Planck's Theory Reexamined

We have conjectured that before the beginning of February 1913 Bohr's theory was, in the sense of its original intention, almost completed and that the consideration of the electron binding played a certain, not trivial, role in those days. If this conjecture is accepted, then Bohr's citation of Planck's papers of 1910–12 ought to be taken seriously. For Bohr indeed declares that in the "first part of the paper the mechanism of the binding of electrons by a positive nucleus is discussed in relation to Planck's theory."[25]

Heilbron and Kuhn's criticism to us consists in this, that Bohr took from Planck's paper only the general idea of the discontinuity of energy, which he could have taken as well, or even better, from Planck's original paper of 1900.[26] We however would like to emphasize that Bohr deals here with the emission of light accompanying the binding of electron, that is, a concrete mechanism of emitting radiation. On the other hand, Planck's original theory of 1900 does not touch the mechanism of emitting radiation. Only the distribution of total energy over individual resonators is considered there. It was in his *Vorlesungen über die Theorie der Wärmestrahlung* of 1906 that Planck first alluded to the mechanism of emitting radiation.[27] But this allusion is merely a vague suggestion that it would be necessary to modify the equation for an oscillator in order to give an electromagnetic foundation of h. It was as late as in 1910–12 that Planck considered the problem in detail.

In 1910, examining Jeans' thorough investigation on the heat radiation, which had shown that the classical theory necessarily leads to Rayleigh's formula, Planck asserted that the equation of motion for an oscillator be modified so that it somehow comprises h.[28] He opposed the theory of light quantum by A. Einstein and J. Stark for the reason that such a theory would imply a retrogression over centuries of the theory of light. He instead proposed to modify the equation of oscillator, leaving the Maxwell equations unchanged, so that the energy of oscillator could change only by an integral multiple of $h\nu$, where ν is the frequency of the oscillator. In making this proposal Planck characterized his position saying that "introducing the quantum of action into the theory, we should be as conservative as possible." But no mathematical formulation was given at this time.

A year later, in February 1911, Planck modified his former view that the energy of oscillator would change discretely by an integral multiple of $h\nu$.[29] According to

[25] N. Bohr, Ref. 7 esp. p. 3.

[26] Heilbron-Kuhn, pp. 268–269, note 145.

[27] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung,* Leipzig, 1906. Esp. footnote on p. 108.

[28] M. Planck, "Zur Theorie der Wärmestrahlung", *Ann. d. Phys.*, **31**, 758–768 (1910); *Physikalische Abhandlungen und Vorträge*, II, Friedr. Vieweg & Sohn, Braunschweig, 1958, pp. 237–247.

[29] M. Planck, "Eine neue Strahlungshypothese", *Verh. d. Deutsch. Phys. Ges.*, **13**, 138–148 (1911); *Phys. Abh.*, II. pp. 249–259.

the former view, argues Planck, the amount of energy that the oscillator would absorb at a time ought to be at least $h\nu$. This will lead to a difficulty when the radiation field is very weak. For the absorption will not occur at all when the radiation is extremely weak. There is still another difficulty. For the short wavelength of radiation, since the smallest amount of absorbed energy $h\nu$ becomes large and the energy density rapidly falls, the time length required to absorb the whole $h\nu$ will greatly increase. Then, if the incident radiation is suddenly shut off by some cause after the oscillator has begun to absorb the radiation, it is no longer possible for the oscillator to receive the whole energy quantum which is needed to maintain its average energy in an equilibrium. Thus Planck decided to return to the view that the oscillator absorbs the radiation energy continuously. But instead he assumed that the emission of energy by the oscillator takes place discretely. The emission of radiation is supposed to take place in the form of energy quantum, independently of the absorption, and according to a law of chance. The energy of an oscillator at an instant is put

$$U = n\varepsilon + \rho; \quad \varepsilon = h\nu, \quad 0 < \rho < \varepsilon,$$

where n is an integer. If one writes the probability that the oscillator emits a single quantum during sufficiently short time dt as $\eta \cdot n \cdot dt$, it is shown that between the average energy of the oscillator $\bar{U}$ and the energy density of the radiation field u_ν, there is a relation

$$u_\nu = \frac{8\pi\nu^2}{c^3}\left(\bar{U} - \frac{h\nu}{2}\right).$$

Now, if there are N similar oscillators, then

$$U_1 = n_1\varepsilon + \rho_1, \quad U_2 = u_2\varepsilon + \rho_2, \ldots, \quad \ldots$$
$$N\bar{U} = U_1 + U_2 + \ldots.$$

The total number of quanta, $P = n_1 + n_2 + \ldots$, is expressed as

$$P = N\left(\bar{U} - \frac{\varepsilon}{2}\right)\Big/\varepsilon,$$

where the average value of $\rho_1, \rho_2, \ldots$ has been assumed to be $\varepsilon/2$. If one assumes here that the entropy corresponding to $\bar{U}$ of the oscillator be given by $S = k \log W$, where W is the number of ways to resolve P into a sum of integers $n_1, n_2, \ldots$, the right formula of radiation is easily obtained in a similar way as in the 1900 paper.

Several months later, July 1911, Planck again modified his theory above.[30] In the preceding theory he considered the probability that an oscillator emits a single quantum $h\nu$. Now he argues that this implies that the elementary domains in the phase space having an equal probability are not represented by the energy quantum ε. The purpose of new modification was to make all the values of integral

30 M. Planck, "Zur Hypothese der Quantenemission", *Sitz. Ber. Preuss. Akad. Wiss.*, Juli, 1911, pp. 723–731; *Phys. Abh.*, II. pp. 260–268.

multiples of ε of the energy of oscillator equally probable. For this purpose Planck assumed that the energy of an oscillator increases continuously but it is radiated out discontinuously, that is, the oscillator radiates out its total energy $n\varepsilon$ with a certain probability η only at each instant when its energy has just reached integral multiples of ε. This theory was developed in full detail in the following year 1912,[31] which we outlined in our previous paper. It should here be noted that the novelty of this theory compared with the previous one consists in its assumption that the oscillator radiates out not a single quantum, but n energy quanta by a single act of emission.

Planck's papers cited by Bohr in the latter's 1913 paper are those of 1910, early 1911, and 1912. It is easily seen that these papers were chosen not arbitrarily but following the development of Planck's theory. In the first 1910 paper Planck posed the problem of the mechanism of emission, in the second paper of early 1911 he introduced the probability into the emission process, and finally in the 1912 paper he developed the formulation in detail by introducing the new hypothesis that an oscillator radiates out its whole energy by a single act of emission. In relation to Bohr's theory, the notion of probability and the assumption of simultaneous emission of more than one quanta are very significant. Bohr in fact states in his 1913 paper: "the essential point in Planck's theory of radiation is that the energy radiation from an atomic system does not take place in the continuous way assumed in the ordinary electrodynamics, but that it, on the contrary, takes place in distinctly separated emissions, the amount of energy radiated out from an atomic vibrator of frequency ν *in a single emission being equal to* $\tau h\nu$, where τ is an entire number, and h is a universal constant."[32] (Italics supplied.) And just at this place he cites three Planck's paper in question. It is, as we have shown above, during mid–1911 to 1912 that Planck's theory took the shape as summarized by Bohr. In two papers of 1900 and 1901[33] Planck only discusses the partition of total energy to each resonator, keeping silence about the mechanism of the emission of light. In the *Vorlesungen* of 1906 the radiation formula is derived in the same manner as in the 1900 and 1901 papers. It was as late as in 1911 that Planck propounded the notion that the emission of light by an oscillator is discrete and probabilistic, and then assumed that not a single but several, all the energy quanta possessed by the oscillator are emitted at a time. Therefore, Planck's theory which Bohr had in mind at his writing the 1913 paper cannot be the original theory of 1900. The conclusion seems inevitable that from Planck's papers of 1911–12 Bohr took the ideas that could not be taken from his earlier papers. We therefore still maintain the view that it was not for the sake of up-to-dateness, as Heilbron and Kuhn

[31] M. Planck, "Über die Begründung des Gesetzes der schwarzen Strahlung", *Ann. d. Phys.*, **37**, 642–656 (1912); *Phys. Abh.*, II, pp. 287–301.

[32] N. Bohr, Ref. 7, esp. p. 4.

[33] M. Planck, "Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum", *Verh. d. Deutsch. Phys. Ges.*, **2**, 237–245 (1900); *Phys. Abh.*, I. pp. 698–706. "Ueber das Gesetz der Energieverteilung im Normalspektrum", *Ann d. Phys.*, **4**, 553–563 (1901); *Phys. Abh.*, I. pp. 717–727.

suggest,[34] but of substantial significance that Bohr cites Planck's papers of 1910–12.

5. Other Contemporary Theories

Bohr referred to Planck's theory in order to discuss the emission of light by the binding of electron in relation to it. Heilbron and Kuhn, however, suggest that in associating the binding of electron with the emission of light Bohr could follow the widespread view of the time that the emission of the spectral line is connected with the ionization of atom.[35] It seems therefore desirable to examine that widespread view. Heilbron and Kuhn especially mention theories of J. J. Thomson[36] and J. Stark.[37]

Thomson supposes that when an ion and an electron recombine to form the original atom, an energy which is equal to the ionization potential is emitted as a pulse of radiation. This pulse is thought to be radiated out classically by the *bremsung* of electron. The wave length corresponding to the maximum of the energy distribution in the pulse is estimated by having recourse to Wien's displacement law. Finding the temperature corresponding to the molecular kinetic energy which is equal to the ionization potential, λ_{max} of Wien's law at this temperature is calculated. The λ_{max} thus obtained is defined as the wavelength of the pulse emitted. According to Stark, the spectral line is produced when an atom loses one of its valence electrons. Stark supposes that to each spectral line corresponds an oscillator and that this oscillator is the element of positive electricity, the *archion* as he calls, which has been combined with the lost electron when the atom was neutral. Both in Thomson's and Stark's conceptions, the radiation is emitted classically, one by a *bremsung* and the other by an oscillation of positive electric charge. In Planck's theory of 1911–12, on the contrary, the oscillation of resonator is separated from the emission of electromagnetic waves. Radiation is assumed to be emitted independently of the oscillation, according to a law of chance. Right at this point Planck is clearly on the side of Bohr. Heilbron and Kuhn see the decisive difference between Planck and Bohr in the equality, according to Planck's theory, of the frequencies of the emitted radiation and the oscillator. But if the stress is laid on the separation of the oscillation of resonator from the emission of electromagnetic waves, Planck's theory certainly implies a novel idea.*

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34 Heilbron-Kuhn, p. 269, note 145.

35 Heilbron-Kuhn, pp. 263–264.

36 J. J. Thomson, "Ionization by Moving Electrified Particles", *Phil. Mag.*, (6) **23**, 449–457 (1912).

37 J. Stark, *Prinzipen der Atomdynamik, II. Die elementare Strahlung*, Leipzig, 1911, esp. §26; "Zur Frage nach dem Träger und dem Sitz der Emission von Serienlinien", *Jahrb. d. Rad. u. Elekt.*, **8**, 231–240 (1911).

* When Hirosige talked with Kuhn, the latter stressed that the break with the classical theory had been evident since Planck first put forward his theory in 1900. With regard to the discreteness of energy, this is certainly true. We however would like to emphasize that for the first time in 1911–12, a mechanism of radiation which was quite alien to the classical electromagnetic theory was proposed by Planck.

The foregoing consideration shows that there is a clear distinction between Planck's theory and other contemporary theories which associate the spectral line with the ionization. Our claim of the influence of Planck's theory, not of the general conception of the time, on the development of Bohr's thought may therefore be justified.

We have thus far regarded Bohr's discussion of the binding of electron in relation to Planck's theory as an actual step of the formation of Bohr's theory. But even when, conceding to Heilbron and Kuhn's argument,[38] we admit that it was for the sake of enhancing the plausibility of the reasoning by which Bohr devised $W=(\tau h/2)\omega$ that he mentioned the relation to Planck's theory, it would still be necessary to give a physical interpretation to the analogy with Planck's theory in order that this analogy would enhance the plausibility of his reasoning. There seems to be no other way than to take the discussion of the binding process seriously if one seeks such an interpretation. As a possible interpretation required, we suggested in our previous paper that Planck's notion of the probabilistic emission of radiation would be related to Bohr's assumption that the frequency of radiation emitted during the binding is equal to $\omega/2$, the average of electron's frequencies of revolution, 0 and ω, in its initial and final orbits. This is of course a conjecture. But the binding of electron seems, as we have stressed earlier, to have been the problem that Bohr was attacking strenously at the biginning of 1913. And, as was shown above, it was not Planck's original theory of 1900 but his later theory of 1911–12 that could afford a model for considering the emission of radiation by the binding of electron.

In conclusion we reaffirm the essential point of our former view on the possible influence which Planck's theory of 1911–12 would have exerted upon the genesis of the Bohr atom.

6. Concluding Remark

Bohr's theory of atomic constitution of 1913 has long been considered to be the theory of atomic spectra which was to initiate a new stage of the old quantum theory. Our foregoing consideration, however, suggests that the Bohr theory was endowd with a kind of duality. Genetically it was a theory of the chemical properties of atom. But immediately after Bohr had become to believe himself to have completed his theory as such, incidentally the Balmer formula entered into the scene. The Balmer formula forced Bohr to devise a notion of vital importance for the subsequent development of the investigation of the atomic structure, that is, the notion of quantum transition. The development of the atomic theory in the 1910's, however, was not achieved only by the investigation of spectra. Rather the consideration of chemical properties continued to play a major part during

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[38] Heilbron-Kuhn, p. 270.

1910's.[39] The problem with which Bohr was first occupied did not die out until about 1921, when he published a comprehensive study of the atomic structure in relation to the periodic table.[40]

[39] S. Nisio, "The Role of the Chemical Considerations in the Development of Bohr Atom Model", *Jap. Stud. Hist. Sci.*, No. 6, 26–40 (1967).

[40] N. Bohr, "The Structure of the Atom and the Physical and Chemical Properties of the Elements", in N. Bohr, *The Theory of Spectra and Atomic Constitution*, Cambridge Univ. Press, London, 1922, pp. 61–126. "Atomic Structure", *Nature,* **107**, 104–107 (1921). "Atomic Structure", Ibid. **108**, 208–209 (1921).