

# Paul Langevin and the Theory of Relativity

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... une montée pleine de tournants imprévus  
et riche de découvertes, en vue de ce sommet  
tout enveloppé en brume...

LANGEVIN (1933)

## I. Introduction—Historical Background

Paul Langevin (1872–1946) is well-known for his exceptional work on magnetism, but his impressive work on relativity<sup>1</sup> has not been sufficiently appreciated. One should not neglect fascinating objects when they are eclipsed by more brilliant objects!

Lorentz, Poincaré and Einstein were historically the main pioneers in special relativity, building the theory to its culmination in 1905. Moreover, other workers participated significantly. Nor did the story end there since theoretical and experimental work had to follow: extensions, applications, interpretations and clarification of special relativity—here was the role of Planck, Minkowski and Langevin! The last major contribution to special relativity<sup>2</sup> by Einstein was made in 1907, by Poincaré in 1906, and Planck in 1908; Minkowski died in 1909 (of appendicitis) and Poincaré died in 1912, while in the succeeding decade Lorentz made valuable contributions only on a topic or two.

The development of special relativity (starting in 1911) was continued mainly by Langevin, who did essentially the most that was possible in special relativity after

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<sup>1</sup> Langevin's collected articles are in his three books: *Oeuvres Scientifiques de P. Langevin* (Paris: Centre Nat. Rech. Sci., 1950), *La Physique depuis vingt ans* (Paris: G. Doin, 1923), *Izbranye Trudy* (Moscow: Izdat. Akad. nauk SSSR, 1960). Bibliographies are in his "Oeuvres" and in *La Pensée*, mai-juin 1947, 82–87.

His obituaries are in *La Pensée*, mai-juin 1947, by A. Cotton (pp. 21–30), A. Einstein (pp. 13–14), P. Le Rolland (pp. 34–40). Best surveys of his work: J. Becquerel, *Le Principe de Relativité et la théorie de gravitation* (Paris: Gauthier-Villars, 1922), pp. 42, 105–111, 243–244; O. Staroselskaya-Nikitina, *Pol Lanzheven* (Moscow: Gos. Izdat. fiz.-mat. lit., 1962), ch. 5–6; Ya. Dorfman, in Langevin's *Izbranye Trudy*, pp. 721–746; Yu. Geyvish, *Pol Lanzheven* (Moscow: Izdat. Akad. nauk, 1955); H. Arzeliès, *Relativistic Kinematics* (New York: Pergamon Press, 1966), pp. 79, 140, 187–189, 240; *idem*, *La Dynamique Relativiste* (Paris: Gauthier-Villars, 1957), Vol. I, pp. 21–22; Vol. II, p. 406; *idem*, *Relativité Généralisée* (Paris: Gauth.-Vill., 1961), Vol. I, p. 347.

<sup>2</sup> Einstein, *Ann. Physik*, 1907, 23: 371–384; *Jahrb. d. Radioakt.* 1907, 4: 411–462; for Poincaré and Planck see C. Cuvaj, *Am. J. Phys.*, 1968, 36: 1103, 1105; 1970, 38: 774–775.

Minkowski and with more clarity and elegance than *any* of the other principal workers in relativity. One may emphasize that he clarified and extended the work of Einstein and Minkowski. Langevin even contributed to the foundations of the theory by his independent pioneering work on the mass-energy connection in 1905. Many of the most puzzling problems of special relativity were either initially solved by Langevin or given their simplest solution by him. The simplicity, originality, and excitement characterizing his work are impressive. To learn relativity, improve one's understanding, and learn of new approaches, one need only turn to Langevin's work.

One may note some interesting parallels between the cases of Langevin and Henri Poincaré (1854–1912), the other leading French contributor to special relativity. I have shown elsewhere<sup>3</sup> that Poincaré's work in relativity had been often misunderstood and the main parts of it neglected in general. There is a unique difficulty in studying their work, originating mainly from incompleteness of original sources: Poincaré's elliptical presentation and Langevin's lost or unpublished work. Both of them exerted no crucial influence on the trend of relativity. This is not too surprising in Langevin's case as he did not publish some of his most important basic work. Langevin's (and Poincaré's) work is interesting *per se* because of its quality and originality regardless of its external influence (or lack of it). Although relativity without Langevin and Poincaré would not have developed in a very different way, both Frenchmen could have achieved special relativity in a *full* version around the turn of the century, had the circumstances been slightly different—this alone is impressive. Otherwise, one should note that Langevin and Poincaré were very different professionally and personally. For example, Poincaré retained the ether-concept up to his death, while Langevin ascribed much less significance to the ether and even gave it up after 1911. In addition, Poincaré and Langevin exerted little influence<sup>4</sup> on each other in relativity.

After 1905 it was also important to make relativity more available for others—or to popularize it; again, Langevin distinguished himself. This was no easy task because of the resistance and slow accommodation of physicists to the new revolutionary theory of Einstein. Even some great scientists did not adjust well.<sup>5</sup> Joliot-Curie referred to the situation as, “le combat de la relativité.”<sup>6</sup> As late as the 1920's

<sup>3</sup> Cuvaj, *op. cit.*

<sup>4</sup> Although Langevin referred to Poincaré as “maître” in his obituary (“H. Poincaré,” *Revue de Métaphys. et de Morale*, 1913, 21: 718), this may have been mainly a term of reverence for the great man (whose books he read and some of whose Sorbonne class-lectures he must have heard probably in 1898–1900) for Langevin later considered his principal teachers or guides to have been Pierre Curie, Marcel Brillouin and J. J. Thomson (Staroselskaya, “Langevin,” p. 21). Langevin and Poincaré travelled together in 1904 to St. Louis (Missouri) to the Congress of Art and Science. Langevin taught for some 40 years at the Collège de France while Poincaré taught mainly at the Sorbonne.

<sup>5</sup> For example, Lorentz favoured the ether, absolute space and time up to his death in 1928; see S. Goldberg, *Am. J. Phys.*, 1969, 37: 990–993; A. Einstein, H. and M. Born, *Briefwechsel* (Munich: Nymphenburger Verlagshandlung, 1969), pp. 264–265, 72.

<sup>6</sup> F. Joliot-Curie, *La Pensée*, mai-juin 1947, 37.

there was frequent confusion and opposition.<sup>7</sup> For example, the French mathematician Paul Painlevé misunderstood relativity and engaged in controversy, but after 1922 turned relativist. Langevin showed his benevolence by his noble treatment of his critics in relativity, such as Painlevé and Picard, who attacked him strongly, while Langevin just had kind words for them<sup>8</sup>. His humor and wit shows in his words to P. Frank: "Painlevé studied Einstein's work very closely, but unfortunately not until after he had written about it. Perhaps he is used to this sequence from politics."<sup>9</sup> Many physicists were confused or ignored important advances; Poincaré's writings on relativity up to his death in 1912 totally ignored Einstein and Minkowski. Even today special relativity can still be puzzling and controversies abound; new contributions and approaches to special relativity are frequent, many such still being published.

Yet, Langevin eagerly adopted relativity and was one of the few to do so—making himself famous and notorious. Moreover, Einstein wrote in Langevin's obituary, originally in French:

It appears certain to me that he would have developed the special theory of relativity, had that not been done elsewhere; for he had clearly perceived its essential aspects.

Langevin had extraordinary clarity and vivacity in scientific thought, together with a sure intuitive vision for the essential points.<sup>10</sup>

At times Einstein even referred to relativity theory as the theory of Langevin-Einstein.<sup>11</sup> Indeed, Langevin's work has a similar simplicity and elegance to Einstein's. In addition, he knew experimental physics well.

In 1922, Jean Becquerel<sup>12</sup> discussed much of Langevin's work up to then and called Langevin "the great initiator and defender of relativistic theories in France." Opponents of relativity called Langevin "the 'apostle' of the new 'religion' [relativity]," to which Becquerel replied that the study of relativity requires not faith or adoration but just an examination of facts.<sup>13</sup> Historians of science in the Soviet Union have paid a lot of attention to Langevin's work in relativity, in particular Staroselskaya-Nikitina. But these latter treatments contain no mathematics and are not thorough enough. As they are written in Russian and not readily available, an English treatment is much needed.

<sup>7</sup> This is also told by A. Metz, *La Relativité* (Paris: Chiron, 1923), with "Préface" by J. Becquerel; P. Frank, *Einstein—His Life and Times* (New York: A. Knopf, 1953), pp. 194, 203.

<sup>8</sup> Staroselskaya, *Lanzheven*, pp. 121–122.

<sup>9</sup> Frank, "*Einstein*," p. 194. Painlevé (similarly to another French mathematician É. Borel) was a man of immense energy and held government positions of great importance.

<sup>10</sup> Einstein, *La Pensée*, mai-juin 1947, 13–14.

<sup>11</sup> Geyvish, "*Lanzheven*," p. 22.

<sup>12</sup> Becquerel, "*Relativité*".

<sup>13</sup> Becquerel, in Metz, *Relativité*.

## II. A Brief Chronological Survey of Langevin's Work in Relativity

In this section I briefly treat chronologically Langevin's contributions to relativity; thereafter three sections elaborate the new space-time, the twin paradox, and mass-energy.

It well pays to ask what contributions were made by the man whom Einstein considered a potential discoverer of the theory of relativity of 1905. Before starting, one should stress that independently of Einstein, in unpublished work, Langevin<sup>14</sup> discovered the mass-energy relationship, giving a more general derivation than Einstein's. Einstein admitted this in a conference in 1922.<sup>15</sup> Otherwise Langevin did not contribute much to the historical foundations of relativity. Einstein also said of Langevin: "... the fruits of his work appear more in the publications of other scientists than in his own ones."<sup>16</sup> Indeed, as will be shown later, much of his work was lost and the remains can be puzzling.

To evaluate the contributions by a contemporary of Einstein it is important to concentrate on his work just before and after Einstein's<sup>16</sup> articles starting in September 1905. Langevin's four earliest works relevant to relativity (February<sup>17</sup> and September<sup>18</sup> of 1904, March<sup>19</sup> and May<sup>20</sup> of 1905) already indicate his talents.

Although Langevin at first believed in the reality of the ether<sup>21</sup> in 1904-5, it was just regarded as the seat of electromagnetic phenomena, not as an entity to be represented mechanically. Later, he came to share Einstein's view of 1905, although in 1911<sup>22</sup> his ether seemed to have some qualities of absolute space because he regarded acceleration as absolute, as shown by electromagnetic waves emitted by an accelerated charge. For example, in 1912<sup>23</sup> in a chapter entitled "properties of ether" he referred to the ether simply in terms of the properties of the electromagnet-

<sup>14</sup> See E. Bauer, *La Théorie de la Relativité* (Paris: L. Eyrolles, 1922) p. 55; O. Costa de Beauregard, *La Théorie de la relativité restreinte* (Paris: Masson, 1949), p. 87; Staroselskaya, *Lanzheven*, p. 118; J. Abelé, *Arch. de Philos.*, 1956, 19: 20; L. de Broglie, *Savants et Découvertes* (Paris: Albin Michel, 1951), p. 259.

<sup>15</sup> M. Morand, *La Nature*, 1922, 50: 319.

<sup>16</sup> Einstein, *Ann. Physik*, 1905, 17: 891-921, 18: 639-641.

<sup>17</sup> Langevin, in H. Poincaré et al., *L'Enseignement des sci. mathém. et des sci. phys.* (Paris: Imprimerie Nationale, 1904), pp. 73-95; reprint in his *La Phys. . . vingt ans*, pp. 424-453, (hereafter called: Langevin, 1904 A).

<sup>18</sup> Langevin, Sept. 1904 St. Louis lecture, *Revue gén. Sci.*, 1905, 16: 257-276, (hereafter called: 1904 B).

<sup>19</sup> Langevin, *Jour. de Phys.*, 1905, 4: 165-182.

<sup>20</sup> Langevin, *Compt. Rend.*, 1905, 140: 1171-1172.

<sup>21</sup> Langevin, 1904 A, 1904 B; *Ann. Chim. Phys.*, 1905, 5: 70-127.

<sup>22</sup> Langevin, *Scientia*, 1911, 10: 47-48. See also T. Hirose, *Jap. Stud. Hist. Sci.*, 1968, 7: 44-45, 49. Langevin thought that the radiation by an accelerated charge had an absolute sense so that one could detect one's acceleration by electromagnetic experiments inside an accelerated system. By 1919 he did not retain these ideas. On this still controversial topic see F. Rohrlich, *Phys. Today*, March 1962, 15: 19-23.

<sup>23</sup> Langevin, "Les Grains d'électricité et la dynamique électromagnétique" (1912 conference); reprint in his *La Phys. . . vingt ans*, pp. 70-170.

ic field without mentioning the ether as basic, absolute, resting, or referring to motion relative to it. This is consistent with relativity, Einstein's ideas, and the modern view.

Already by February of 1904 he recognized<sup>24</sup> the profound nature of Mach's criticism of Newtonian ideas, according to which, for example, for absolute space one should substitute the relationship of a body to other bodies in the universe, for only so does the motion of the body become observable. Langevin realized in 1904 the approximate nature of old mechanics and the obscurity of its foundations. According to Mach, an old theory (mechanics) need not be the basis of all physics (just because it is historically the oldest theory), for it may well be an insufficient theory.<sup>24</sup> Langevin considered electromagnetism to be more fundamental, accurate and appropriate as the basis of physics. However, he wisely saw the limitations of the electromagnetic description of the world, as in the case of gravitation.<sup>25</sup> He even believed mechanics to hold as a first approximation valid for macroscopic bodies (but not for enormous distances), neglecting radiation, for small velocities; so that it would not be valid for the electron. This showed quite a lot of insight before the work of Lorentz in May 1904,<sup>26</sup> Poincaré in 1904–5, and Einstein—before relativistic and quantum mechanics.

It is interesting to note Langevin's suggestion<sup>27</sup> of unknown forces holding an electron in equilibrium (against the mutual electrostatic repulsion of the charge) corresponding to a new kind of energy perhaps connected to gravitation. This may have influenced Poincaré who heard these ideas in Langevin's lecture at the 1904 Congress in St. Louis, so that he suggested the hypothetical "Poincaré stresses" in 1905.<sup>3</sup> In March 1905,<sup>19</sup> Langevin gave an elegant treatment of the velocity and acceleration fields of an electron, (introducing these names and this separation of the fields), and its energy and radiation. In May, he had the first satisfactory explanation of the Trouton-Noble experiment;<sup>28</sup> in fact other types of explanations have not been successful even up to the present, as was recently pointed out in a long-needed critical article by Butler.<sup>29</sup>

A curious fact is the relatively small number of publications by Langevin in the years 1906–1910 (inclusive); moreover, no publication on relativity. After having published 13 articles in 1905, he had apparently only six in 1906–10, and six in 1911–12. Was he "resting" after the busy year of 1905 or concentrating on his lecturing?

After this gap of several years, in which he did important unpublished work in relativity (more on mass-energy), appear Langevin's two articles of 1911,<sup>30</sup>

<sup>24</sup> Langevin, 1904 A, pp. 437–451.

<sup>25</sup> Langevin, 1904 B, p. 269.

<sup>26</sup> H. Lorentz, *Kon. Akad. Wet., Proc.*, 1904, 6: 809–831.

<sup>27</sup> Langevin, 1904 B, pp. 267–268.

<sup>28</sup> Appendix.

<sup>29</sup> J. Butler, *Am. J. Phys.*, 1968, 36: 936–941.

<sup>30</sup> Langevin, *Scientia*, 1911, 10: 31–54, (hereafter called: 1911 A); *Bull. Soc. Franç. Philos.*, 1912, 12: 1–46, (hereafter called: 1911 B).

(addresses at a congress of philosophy in Bologna and at a meeting of the Société française de philosophie in Paris), where he distinguished himself in his philosophical and physical treatment of the problem of time by clarifying<sup>31</sup> the concepts of causality, proper time, space-like and time-like intervals, and space-time in general. For example, he first established the unusual result that the interval in *space-time* is *maximum*, contrasted to that (of minimum) in *space* for uniform straight-line motion between two points. At this time Einstein's problem of twins (of 1905) and the relativistic explanation of the Michelson experiment also received the first thorough treatment, by Langevin.

In 1913,<sup>32</sup> Langevin even predicted the possibility of an enormous atomic energy release (now familiar in reactors and atomic bombs). He also applied  $E = mc^2$  to nuclei in explaining deviations from Prout's law of integral atomic "weights." In 1921–37 he first applied general relativity in detail to rotating discs,<sup>33</sup> treating Ehrenfest's paradox, concerning the inapplicability of Euclidean geometry to a rotating disc, and explaining Sagnac's experiment.<sup>34</sup> He was also greatly interested in the basis of general relativity, which he illustrated by ingenious examples.<sup>35</sup> In 1926,<sup>36</sup> before Thomas' 1927<sup>37</sup> work on precession, Langevin obtained Thomas' results. He may be judged also by his two lectures in Einstein's presence, in 1922 and 1931,<sup>37</sup> during the visits of Einstein to Langevin in Paris. (They had met first at the Solvay congress in Brussels in 1911). An excellent talk before such an audience is no small achievement.

Among his late works on relativity were a lecture in 1928<sup>38</sup> in Tbilisi (Soviet Georgia), a conference on relativity that he organized in Paris in 1932, and his last big lectures in 1933 and 1938.<sup>39</sup> In those years he was interested in solar and astro-

<sup>31</sup> Arzeliers, *Rel. Kinem.*, p. 140.

<sup>32</sup> Langevin, *Jour. de Phys.*, 1913, 3: 553–591, reprint in his "Oeuvres," pp. 397–426, (hereafter called: 1913).

<sup>33</sup> Langevin, *Comptes Rendus*, 1921, 173: 831–834; 1935, 200: 48–51, 1161–1165; 1937, 205: 304–306.

<sup>34</sup> Langevin, "Le Principe de relativité," *Bull. Soc. Franç. Électriciens*, 1919, 9: 601–639; *Bull. Scient. Étudiants Paris*, 1922, no. 2: 2–22, (hereafter called: 1922); and later articles.

<sup>35</sup> See A. Sommerfeld, *Atombau und Spektrallinien* (5th ed., Braunschweig: F. Vieweg, 1931), Vol. I., pp. 708–711; transl. as *Atomic Structure and Spectral Lines* (New York: E. Dutton, 1933), pp. 662–667; Reale Accademia d'Italia, *Convegno di Fisica Nucleare* (1931), 137–141.

<sup>36</sup> L. Thomas, *Phil. Mag.* 1927, 3: 1–22.

<sup>37</sup> Langevin, 1922: "L'Oeuvre d'Einstein et l'astronomie," *Bull. Soc. Astron. France*, 1931, 45: 277–297, (hereafter called: 1931).

<sup>38</sup> Langevin, "La Structure des atomes et l'origine de la chaleur solaire," *Bull Univ. Tiflis*, 1930, 10: 01–05.

<sup>39</sup> Langevin, respectively: "La Relativité" *Actualités sci. industr.* (Paris: Hermann, 1932), no. 45; "L'Évolution de la science électrique depuis 50 ans," Soc. Franç. Electriciens, *Célébration du cinquantenaire* (1933), 131–154, (hereafter called: 1933); unpublished lecture, mentioned in *La Pensée*, mai-juin 1947, 58.

nomical implications of relativity. His last works<sup>40</sup> were the notes (1935–7)<sup>33</sup> on the Sagnac experiment.

Nobody could present the new (or old) ideas of relativity in a better way, verbally or mathematically than Langevin. At times he even did so better than Einstein. Langevin's early contact with relativity before 1905 gave him knowledge of the difficulties of the classical theory. As he had witnessed the historical development he could authoritatively expound it and appreciate the breakthroughs. His insight and enthusiasm resulted in a fresh approach not only in his contributions to relativity for many years (1904–1937) but also in his many popular presentations. He emphasized remarkable examples, analogies, the general unity and symmetry introduced by relativity, and many interesting experimental results and hypotheses alternative to the usual interpretation. According to Langevin, the unity and harmony in the new theory resulted from the concepts of space-time, energy-mass-momentum, combined electric and magnetic fields, and the synthesis of optics and mechanics or light and matter (and even the particle-wave synthesis of quantum theory).

Another impressive characteristic of Langevin's work was his insight into misconceptions, wrong habits and illusions in physics (and elsewhere). He gave a deep and basic analysis of the classical crisis, clearly pointing out the reasons for the difficulties in the old physics, while some did not even know they existed. This is of great importance for one of the most difficult tasks of physics is the overcoming of such obstacles. One can hope to infer from such past lessons where and how future breakthroughs might occur, how to become more cautious, and how to avoid analogous illusions.

### III. The New Space-Time

To introduce the basic vocabulary for this section one may turn to Langevin's interesting definitions and interpretations (1911) of the basic concepts of relativity. Space is "... a slice of the universe at a given time. ..." <sup>41</sup> Time is the ensemble of events succeeding at a point. The universe is the ensemble of all events, *i.e.* the ensemble of all space-time. Time defined by the optical or electromagnetic method agrees with the time of the Lorentz group. Mechanical, biological, chemical and other ways, even if their accuracy is comparable to the optical, must yield the same time measurement—by the principle of relativity—admitting the Lorentz group for all those phenomena.

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<sup>40</sup> Completing the list of his relevant works are: *Soc. Franç. phys., Procès-verbaux*, 2 déc. 1921, 77–79; 16 déc. 1921, 97–98; *Bull. Soc. Franç. phys.* 1920, **138**: 5 (reprint in *Oeuvres*, pp. 427–435); *Bull. Soc. Franç. Phil.*, 1922, **17**: 93–96, 98–99. For his formulation of electrodynamics see R. Debever et J. Géhéniau, *Acad. Roy. Belg., Bull. classe sci.*, 1955, **41**: 346–355. See also Langevin's article on gravitation in *Ann. de Phys.*, 1942, **17**: 265 (reprint in *Oeuvres*, pp. 673–680).

<sup>41</sup> Langevin, 1911 B, p. 10.

### A. A Derivation of the Lorentz Transformation

By a simple physical demonstration<sup>42</sup> Langevin obtained in an original fashion the equations of the Lorentz group using only the principle of relativity (in considerations of symmetry), the identity of the speed of light in opposite directions (a particular case of isotropy of propagation of light in all Galilean systems), and homogeneity of space. He considered a "light-clock" as a box of perfectly reflecting walls where a light-signal successively reflects on opposite faces A and B. In the laboratory frame the Lorentz-contracted length of the box is  $L$  and its speed is  $v$ , both along the  $x$ -axis normal to the faces A and B.

When light travels from A to B, as its speed is constant, one finds  $t_{AB} = L/(c - v)$ , from  $ct = vt + L$ ; but for the reverse case  $t_{BA} = L/(c + v)$ , from  $ct + vt = L$ . The period of the clock is  $T = L/(c - v) + L/(c + v) = 2Lc/(c^2 - v^2)$ , the time required for a round-trip of the signal. At  $t = 0$  A is at  $x = 0$  and the signal is then emitted at A. Let the event  $E_n$  be the  $n$ th reflection at B with  $x, t$ . Preceding it are  $n$  trips of the light signal to the right and  $(n - 1)$  returns to the left. The event at B is given by

$$t = nL/(c - v) + (n - 1)L/(c + v) = [(2n - 1)Lc + Lv]/(c^2 - v^2),$$

$$x = L + vt.$$

Observers moving with the clock find the same light-speed  $c$  but may use different units, such that  $T' = 2L'/c$ . A may be chosen at  $x' = 0$ , and  $E_n$  has  $x', t'$  as it occurs at B:

$$t' = nL'/c + (n - 1)L'/c = (2n - 1)L'/c,$$

$$x' = L'.$$

By homogeneity of space  $L'/L = \alpha$  is a constant fixed by the choice of units and  $v$ . Then, using  $(2n - 1)Lc = (c^2 - v^2)t - Lv = (c^2 - v^2)t - v(x - vt) = c^2t - vx$ , from above follows

$$x' = L' = \alpha L = \alpha(x - vt),$$

$$t' = \alpha(2n - 1)L/c = \alpha(t - vx/c^2);$$

or inversely, defining  $\alpha' \equiv [\alpha(1 - v^2/c^2)]^{-1}$ ,

$$x = \alpha'(x' + vt'),$$

$$t = \alpha'(t' + vx'/c^2),$$

By the principle of relativity, the two systems are equivalent; if both choose the same units then the expressions relating the two systems must be symmetrical, except for the sign of  $v$ , which restriction can be avoided by placing the axes in opposite senses in the two systems. The coefficients of the  $x$  and  $x'$ , or of the  $t$  and  $t'$  equations must be equal, so that  $\alpha' = \alpha$  or  $\alpha^2 = (1 - v^2/c^2)^{-1} = \gamma^2$ , and so  $\alpha = \gamma$ . This gives the usual Lorentz transformation equations.

<sup>42</sup> Langevin, in E. Bauer, "Cinématique de la relativité," *Actual. sci. ind.* (1932), no. 40.



Then follow  $L'/L = \alpha = \gamma$  or  $L = L'(1 - v^2/c^2)^{1/2}$  and  $T/T' = \gamma^2 L/L' = \gamma$ . Hence, the laboratory observers find that the moving box is shorter by the  $\gamma^{-1}$  factor than  $L'$  measured by the observers in the rest-system of the box.  $T' = (1 - v^2/c^2)^{1/2} T$  shows that there is a shorter time interval  $T'$  between two successive reflections by the same mirror (seen in the same place) for its own observers, than  $T$  obtained by observers with respect to whom the box moves and the two reflections occur in different positions separated by  $vT$ .

Langevin also showed<sup>43</sup> that there is no contraction perpendicular to the motion by the example of two parallel square wire frames approaching each other along the line normal to their planes. A lateral contraction of the moving frame, considering the other one fixed, would mean that it would pass through the fixed one on meeting it—this would be seen in both reference systems. But the “moving” observer may consider himself at rest and the other as moving; there is a contradiction.

### B. *Space-like and Time-like Intervals*

In the articles “L’Évolution de l’espace et du temps” and “Le temps, l’espace et la causalité dans la physique moderne,” appearing in 1911<sup>30</sup> Langevin presented new interpretations of the Poincaré-Minkowski concepts of space-time and of Einstein’s theory. Minkowski developed in 1907–8 Poincaré’s<sup>3</sup> (1905) mathematical interpretation of relativity in terms of four vectors and invariants such as  $ds^2 = dl^2 - c^2 dt^2$ . Moreover, he introduced the closely related concepts and names of proper time and space-like or time-like vectors. His treatment was brief, with few physical interpretations. We owe to Langevin the further development of our modern views. Let us outline his treatment.

An interesting, apparently paradoxical result of the new kinematics is that two observers in relative movement can determine two events to be in one or a reversed order of succession except when the events are in causal order. In the old theory a change in reference system could not reverse the order because one event could modify the conditions of the other event and hence there could be a causal connection no matter what be the distance of separation in space, since the old hypothetical limiting speed of possible signals was infinite and could instantaneously cause events over distance. The time interval  $\Delta t$  between two events was considered the same for all observers for any relative movement, but the space interval  $\Delta l$  depended on observers, as when two objects fall through a hole in a moving train’s, at the same place in the train, whence  $\Delta l' = 0$ , but hit the grounds at different points separated by  $\Delta l = v\Delta t$  (in terms of the speed of the train and the time interval between their fall). Only for simultaneous events could  $\Delta l$  be absolute e.g. the length of a rod. For two non-simultaneous events separated by  $\Delta l$  succeeding one another by the time  $\Delta t \neq 0$ , one could always find a reference system with speed  $v = \Delta l/\Delta t$  with respect to the original system to obtain coincidence in *space* of the two events. But

<sup>43</sup> See E. Bauer, “*Relativité*,” p. 29.

a choice of reference system was not possible to give a coincidence in *time* ( $\Delta t = 0$ ) because  $\Delta t$  is absolute, the same in all systems.

Thus, in contrast to relativity, classical kinematics showed an asymmetry, since  $\Delta t$  was an invariant while  $\Delta l$  depended on the observer's reference frame. In relativity there is only one case where a change in reference system has no effect, namely absolute or double coincidences in space and time ( $\Delta l = 0$ ,  $\Delta t = 0$ ), e.g. a collision. In 1911 Langevin<sup>44</sup> recognized these "invariant simultaneities"; Einstein considered them in 1905<sup>45</sup> only incompletely. Langevin emphasized that Einstein was guided to general relativity by realizing these coincidences to be invariant also for accelerated observers.<sup>46</sup> As all our experiences and sensations are based on such absolute coincidences and as science is founded on such experiences, its laws must have a significance independent of systems of reference with any motion, and so must be generally covariant.

In relativity, for any two events, both  $\Delta l$  and  $\Delta t$  change in general. It will be shown to be important and convenient to class events into two categories for which space and time respectively play symmetrical roles.

*Firstly*, there are space-connected four-events with  $|\Delta l| > c|\Delta t|$  (e.g.  $\Delta t = 0$ ,  $\Delta l > 0$ ) in every uniformly moving reference frame, so that this property has an absolute sense. They are distant enough in space so that one always occurs before a light signal from the other one can arrive. The order of succession can be reversed for some moving observers since  $\Delta t$  is not absolute here. The events are not causally related for they cannot be informed of each other (except by signals with speed greater than  $c$ ), being independent of each other for they have no defined order in time. They cannot belong to the same world-line or same part of matter or of a being. Now  $\Delta l \neq 0$  in every reference frame but one can be found where  $\Delta t = 0$  so that the events are simultaneous, in which case  $\Delta l$  is minimum and will be longer for other frames moving with respect to this one. (This does not signify a preferred frame or asymmetry.) The invariant  $I = (\Delta s)^2 = (\Delta l)^2 - c^2(\Delta t)^2$ , with  $I > 0$  since  $\Delta l > c\Delta t$  here, represents the interval (by definition) between the two events (not connected by a world-line). It gives  $I = (\Delta l)^2$  for  $\Delta t = 0$  in one system, but for  $\Delta t \neq 0$  in other systems, one has  $(\Delta l)^2 = I + c^2(\Delta t)^2 \geq I$  showing that  $\Delta l$  is minimum (equal to  $I$ ) in the system for which the events are simultaneous. Langevin in this fashion arrived at a novel interpretation of the Lorentz contraction.

Langevin gave an interesting simple example for space-connected events. A point-source of light at the origin  $O'$  of its rest frame  $S'$  moves with speed  $v$ , parallel to  $x$  (Fig. 1). An observer  $S$  at rest sees the source move and the wave emitted at his origin  $O$  as a spreading sphere of light with radius  $R = ct$  with center at  $O$ . During the time  $t$  the source moved to the right by  $d = vt$ .  $S$  receives the wave at  $M, N$  (both equidistant from  $O$ ) simultaneously. For  $S'$  moving with the source,

<sup>44</sup> Langevin, 1911 A, p. 41.

<sup>45</sup> Einstein, *Ann. Physik*, 1905, 17: 893.

<sup>46</sup> Langevin, 1922, p. 17. See Einstein, *Ann. Physik*, 1918, 55: 241-244.

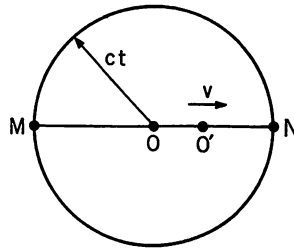


Fig. 1. A spherical light-wave emitted by a moving source and defining two space-connected events M, N.

N receives the signal before M because the distance for the light wave to travel is less. An observer  $S''$  moving in the opposite direction to  $S'$  will have the signal at M first *i.e.* a reversed order from  $S'$ . For the two events at M, N,  $\Delta l > c\Delta t = 0$  since  $\Delta l = x_N - x_M = 2ct$  and  $\Delta t = t_N - t_M = 0$  for S. Using Lorentz transformations and  $\Delta t = 0$  one has  $\Delta t' = t'_N - t'_M = -\gamma v \Delta x / c^2 < 0$ ,  $\Delta t'' = \gamma v \Delta x / c^2 > 0$  and  $\Delta x' = \Delta x'' = \gamma \Delta x$  (where  $\Delta x = \Delta l$ ). Combining gives  $|\Delta x'| = |c^2 \Delta t' / v| > c \Delta t'$  and  $|\Delta x''| = |c^2 \Delta t'' / v| > c \Delta t''$  checking the space-connected property in every reference system. This example also shows how the isotropic propagation of light gives different relative time.

Langevin was led to limiting the propagation of causality to speeds up to  $c$  by considering space-like events. Einstein in 1907 and Langevin rejected the perfectly rigid body requiring elastic waves of infinite speed and also showed that if two events are linked by signals with  $v > c$  then for some moving observers effect would precede cause, so that one could “telegraph into the past.”<sup>47</sup> As Langevin stressed the unity of the concepts of biological, mechanical, chemical, and optical time, for agreement with relativity, the upper limit of speeds was to apply to all phenomena and in general. It has been recently shown, especially by Bilaniuk and Sudarshan, that, although this *upper* barrier holds for ordinary particles, it is meaningful to consider particles with  $v > c$ , for which  $c$  is a *lower* limit (and also that in such a theory communication with the past is still ruled out). These views should be encouraged by Heaviside’s statement: “The moral is—don’t be afraid of infinity.”<sup>48</sup>

*Secondly*, one has *time-connected* events with  $|\Delta l| < c|\Delta t|$ , or near enough events, so that one event is produced after the reception of a light signal emitted at the occurrence of the other event. Again  $\Delta l < c\Delta t$  has an absolute significance (in every frame). One finds exact correspondence to space-connected events by interchanging space and time in the discussion above. There is an asymmetry in time between the two events. A causal connection may exist here since the time order is fixed,

<sup>47</sup> Langevin, 1911 A, p. 44; See Einstein, *Ann. Physik*, 1907, 23: 371–384.

<sup>48</sup> O. Heaviside, *Electromagnetic Theory* (New York: Dover Publ.), Vol. II, 535 (1899); see G. Lee, *Oliver Heaviside* (London: Longmans, Green and Co., 1947), p. 21.

has an absolute sense, and could not be reversed by a change of reference system unless the relative speed of the reference frames exceeds  $c$ . The events can belong to the same world-line of a piece of matter or a living being. One can obtain  $\Delta l = 0$  for a suitable frame, but  $\Delta t = 0$  is impossible if  $\Delta l \neq 0$ , ruling out a reversal of time order from  $\Delta t > 0$  to  $\Delta t < 0$  by changing frame. Here  $I = (\Delta l)^2 - c^2(\Delta t)^2 < 0$  and from  $c^2\Delta t^2 = \Delta l^2 + |I|$ , with  $\Delta l = 0$ ,  $\Delta t$  is minimum for the proper observer and is the proper time interval in the rest-frame following the piece of matter. In any other moving frame the events will be separated by a larger time because they will not be seen space-coincident as in the proper frame. Hence, the symmetry with space-connected events is complete.

The Michelson<sup>49</sup> experiment is an example of time-connected and space-connected events. In 1911 Langevin<sup>50</sup> gave the full relativistic explanation of it and his treatment is the best one I know. This was possible only after 1905, but this treatment had been neglected by others. He had two viewpoints (both relativistic): the aspect of the interference pattern is unchanged upon rotation of the apparatus by  $90^\circ$ ; considered in 1) the rest frame of the apparatus (obviously—for one is effectively at rest); 2) a frame, relative to which the earth is moving—by isotropy of propagation of light and the Lorentz contraction. It is surprising that earlier or later treatments ignore the first (instructive) viewpoint, (showing only the result demanded by the old ether theory) and rarely include the Lorentz contraction in the calculations, so that the reader may not become convinced of the relativistic outcome (or null-result) of the experiment, since such explanations are not really relativistic, but present the ether viewpoint.

It may be appropriate to mention now Langevin's insight in history of relativity, when he realized that Michelson's experiment was not essentially important or necessary for the development of relativity. This agrees with the recent researches of Holton.<sup>51</sup> Langevin's view is evident from the following quotes in two of his lectures in the presence of Einstein:

All the detour made by the Michelson experiment could have been avoided if one had had confidence in the equations of electromagnetic theory as representing all the electromagnetic experiments and that the property of these equations of preserving their form for certain transformations represents the experimental fact of relativity. One would have seen that these results imply a certain kinematics which is not that of absolute time.

... The Michelson experiment is not an isolated experiment upon which one then built a whole system a little in the air; it is only an extremely precise

<sup>49</sup> Michelson's explanations were not good: *Amer. J. Sci.*, 1881, **22**: 120–129 (with an error in transverse time); (with E. Morley), 1887, **34**: 333–345; *Studies in Optics* (Univ. of Chicago Press, 1927) (not relativistic).

<sup>50</sup> Langevin, 1911 B, pp. 11–17.

<sup>51</sup> G. Holton, *Isis*, 1969, **60**: 133–197; see also Becquerel in Metz, *Relativité*; A. d'Abro, *Evolution of Scientific Thought* (New York: Dover Publ., reprint of 2nd. ed. from 1949), p. 147; T. Hirose, *Jap. Stud. Hist. Sci.*, 1965, **4**: 121.

verification of the consequences of the . . . electromagnetic theory, based . . . upon the whole ensemble of electromagnetic phenomena.<sup>52</sup>

### C. The Proper-Time Interval

Langevin in 1911<sup>53</sup> first established the peculiar<sup>54</sup> relativistic result of *maximum* interval  $\Delta s = \int ds$  in space-time between two time-like four-events, for uniform motion in a straight line connecting them, when compared to non-uniform motion. B. Russell called this the "law of cosmic laziness."<sup>55</sup> By contrast, in ordinary three-space geometry  $\Delta s$ , the interval in space is *minimum* for inertial motion between two points in space. Let us review Langevin's mathematical treatment<sup>54</sup> of 1919, which is equivalent to his qualitative conclusions of 1911. The Lorentz-invariant interval  $ds$  or  $\Delta s$  in space-time is obtained from  $ds_1^2 = dx_\mu dx_\mu = dx^2 - c^2 dt^2$  or more conveniently for time-like intervals from  $ds_2^2 = c^2 dt^2 - dx^2 = c^2 dt'^2 - dx'^2$ . Consider a particle moving with velocity  $v$  in the stationary frame  $S$ . Two events on its world-line occur at the same place for an observer  $S'$  tied to the particle ( $x' = \text{constant}$  vector,  $v' = 0$ ) but not so for others. Then  $dx' = 0$  and  $dx = v dt$  give  $ds_2^2 = c^2 dt'^2 = c^2(1 - \beta^2)dt^2$ , (where  $\beta = v/c$ ), or in terms of the invariant proper time  $ds = c d\tau$ , where  $d\tau = dt' = (1 - \beta^2)^{1/2} dt$ . For two distant four-events  $A$  and  $B$   $\Delta\tau = \tau_B - \tau_A = \int_{t_A}^{t_B} (1 - \beta^2)^{1/2} dt$  and the time-interval  $\Delta t = t_B - t_A$  (and also the space-interval) is fixed for the stationary observer in  $S$  (where  $\Delta t > \Delta\tau$ ). To this correspond different  $\Delta\tau$  and  $\beta(t)$  depending on how  $S'$  moves between<sup>56</sup>  $A$  and  $B$ , using different world-lines. Non-uniform motion (in direction or magnitude of velocity) compared to uniform motion will result in a smaller  $\Delta\tau$  for as  $\beta$  increases to keep  $\Delta t$  fixed on the longer path or world-line  $(1 - \beta^2)^{1/2}$  and so  $\Delta\tau$  will decrease. The maximum  $\Delta\tau$  (for our given  $\Delta t$ ) will be for uniform motion in a straight line, *i.e.* for a force-free particle moving along a geodesic in four-space-time. Hence, Russell's name is suitable, since a particle left alone would take the trip as slowly as possible, by its own clocks. The more one deviates from a straight line the less will be  $\Delta\tau = \Delta t'$ . Thus, there is a contrast between the properties of space-time (maximum) and three-space (minimum) regarding the interval  $\Delta s$  between two events. The extremum property for different paths for both cases may be represented by  $\delta \int ds = 0$  giving a maximum and a minimum respectively.

<sup>52</sup> Langevin, 1922, pp. 11–12; see also 1931, p. 283.

<sup>53</sup> Langevin, 1911 A, pp. 48–49; 1911 B, pp. 26–27.

<sup>54</sup> Arising from a minus sign in  $ds^2$ ; see also A. Eddington, *Space, Time and Gravitation* (Cambridge Univ. Press, 1920), pp. 70–71, 75; E. Taylor and J. Wheeler, *Spacetime Physics* (San Francisco: Freeman & Co., 1963), Sect. 5–6; M. von Laue, *Phys. Zeitschr.*, 1912, 13: 118–120.

<sup>55</sup> B. Russell, *ABC of Relativity* (2nd ed., London: G. Allen and Unwin, 1958) p. 78 (1st ed., 1925); also C. Durell, *Readable Relativity* (New York: Harper Torchbooks, 1926), pp. 94–97.

<sup>56</sup> Laue, *op. cit.*, in replying to objections stressed that here is an emphasis on one direction between A and B because of our choice of A and B, but not by the laws of nature, so that isotropy of space still exists.

Another instructive example of the use of the invariant  $ds$  is Langevin's general relativistic treatment (1921, 1935)<sup>57</sup> of a rotating coordinate system of a disc (with constant  $\omega$ ). This method was neglected by others:<sup>58</sup> Weyl gave only  $ds^2$  in the form below; Silberstein just indicated that  $ds = 0$  should be used for the explanation of the Sagnac experiment.<sup>28</sup> Hence, Langevin first<sup>59</sup> treated by this theory the space and time on a rotating disc, defining a local time, and treated Ehrenfest's paradox (1909).<sup>60</sup>

The primed (rotating system  $S'$ ) and unprimed (laboratory  $S$ ) polar coordinates, with respect to the center of rotation, are related by

$$\theta = \theta' + \omega t, r = r', t = t',$$

or,

$$x = x' \cos \omega t - y' \sin \omega t, \quad y = x' \sin \omega t + y' \cos \omega t,$$

(where  $t$  is the laboratory time of the fixed center), all justified to first order in  $R\omega/c$  since dimensions and clocks change only to second order. The invariant  $ds^2 = c^2 dt^2 - dl^2$ , with  $dl^2 = dr^2 + r^2 d\theta^2$ , becomes

$$ds^2 = (c^2 - \omega^2 r^2) dt^2 - 2\omega r^2 d\theta' dt - dl'^2,$$

or, after dropping primes, with the cross-term as  $-2\omega(xdy - ydx)dt$ ,  $r^2 = x^2 + y^2$  and  $dl'^2 = dx^2 + dy^2$ . Because of the  $d\theta dt$  term, there is anisotropy in the propagation of light, whose speed varies between  $c - \omega r$  and  $c + \omega r$  to first order. The use of  $t'$  does not permit a decomposition into space and time in  $S'$ . To obtain isotropy or  $ds^2 = c^2 d\tau^2 - d\sigma^2$ , he introduced in  $S'$  a non-uniform local time  $\tau$  as

$$d\tau = (1 - \omega^2 r^2/c^2)^{1/2} \left( dt - \frac{\omega^2 r^2 d\theta}{c^2 - \omega^2 r^2} \right),$$

or  $d\tau = dt - \omega r^2 d\theta/c^2$  to first order, and non-Euclidean geometry, or<sup>61</sup>

$$d\sigma^2 = dr^2 + \frac{r^2 d\theta^2}{1 - \omega^2 r^2/c^2}.$$

This shows in a natural way a larger circumference  $2\pi r(1 - \omega^2 r^2/c^2)^{-1/2}$  of a circle about the origin measured in  $S'$ , or Ehrenfest's paradox which indicated the first non-Euclidean effect in relativity. As  $d\tau$  is not an exact differential, one cannot define  $\tau$  as a common time to all observers for it depends on the path traversed. Langevin applied these results to Sagnac's experiment<sup>28</sup> and similar ones, involving light-rays or  $ds^2 = 0$ .

<sup>57</sup> Langevin, *Comptes Rend.*, 1921, 173: 831-834; 1935, 200: 48-51; see also Becquerel, *Relativité*, pp. 76-80, 243-244; A. Metz, *J. de Phys.*, 1952, 13: 224-238.

<sup>58</sup> H. Weyl, *Raum-Zeit-Materie* (3rd ed., Berlin: Springer, 1919), p. 190; L. Silberstein, *J. Opt. Soc. Amer.*, 1921, 5: 291-307.

<sup>59</sup> see Arzelès, *Rel. Kinem.*, p. 240; *Rel. Gén.*, Vol. I, pp. 32, 347.

<sup>60</sup> P. Ehrenfest, *Phys. Zeitschr.*, 1909, 10: 918.

<sup>61</sup> For interpreting the case  $\omega r \gg c$  see B. Laurent, *Am. J. Phys.*, 1970, 38: 492.

#### D. Thomas Precession

In a simple way, Langevin obtained the Thomas precession, another puzzling relativistic phenomenon. In 1931,<sup>62</sup> Sommerfeld presented these ideas from an unpublished lecture of 1926 in Zürich by Langevin. Thomas<sup>63</sup> obtained his results in 1927 by different considerations, which lack the clarity of Langevin's direct approach. The only details Sommerfeld gave about Langevin's work were that he obtained the  $1/2$ -factor and considered the precession as arising from the rotation accompanying two successive mutually perpendicular Lorentz transformations. Using Pauli's personal notes of Langevin's lecture Sommerfeld presented this by using his method<sup>64</sup> of geometric interpretation of space-time rotations of Minkowski in the complex four-space, resulting in a compact and remarkably quick solution.

Sommerfeld had been near the solution in 1909 when he had treated most of these later ideas, but at that time he did not consider and compute the precession. The Langevin-Sommerfeld method and results are equivalent to those of Stephenson and Kilmister, using only the algebraic form of Lorentz transformations, or Zatskis following the latter authors and using the matrix form of the transformations.<sup>65</sup> In fact Langevin's method was apparently similar to that of Stephenson-Kilmister. These methods are the most direct ones in literature and involve no approximations until the very last step, while Thomas and others started with approximations and less convenient reference frames. It is desirable to advertise these methods because they have not found sufficient publicity *e.g.* the extraordinary encyclopedic work by Arzeliès<sup>64</sup> overlooked them.

#### IV. The Twin Paradox

Using Minkowski's basic concepts Langevin first treated<sup>66</sup> fully Einstein's 1905 problem of the asymmetrical aging of two space travellers or clocks (sometimes called "Langevin's travellers"), where one separates from another "fixed" one and later returns. Applying the previous results one finds that the fixed twin experiences a longer time interval ( $\Delta t$ ) than the accelerating one ( $\Delta t'$ ). Moreover, of two bodies that meet first, then separate and finally again meet, the least aging results for one that deviates most from uniform motion, or accelerates most. Langevin concluded wittily: "... one could prevent aging by going to promenade."<sup>66</sup> For example, two radioactive pieces can be used as twins; the "vagabond" piece should age less and be less decayed than the fixed one in the laboratory. Retarded aging in humans is an amusing (but still impractical) consequence. Langevin emphasized

<sup>62</sup> A. Sommerfeld, *Phys. Zeitschr.*, 1909, 10: 826–829.

<sup>63</sup> G. Stephenson and C. Kilmister, *Special Relativity for Physicists* (London: Longmans, Green & Co., 1958), pp. 30–31; H. Zatskis, *J. Franklin Inst.*, 1960, 269: 268–273.

<sup>64</sup> Arzeliès, *Rel. Kinem.*, pp. 173, 198, 201; and also G. Holton, "Resource Letter on Special Relativity," *Am. J. Phys.*, 1962, 30: 462–469.

<sup>65</sup> see also Arzeliès, *Rel. Kinem.*, pp. 187–189; É. Borel, *Space and Time* (New York: Dover Publ., reprint of 1922 ed.), pp. 26, 144.

<sup>66</sup> Langevin, 1922, p. 15.

the unity of different concepts of time; applied to biological time this justifies the conclusion for the twin problem as "... we are ourselves clocks."<sup>67</sup>

#### A. *Langevin's Space Twins*

It is interesting to note that continuing a traditional French interest in rockets (originating in the books of Jules Verne around 1865) Langevin apparently introduced them into physics starting in 1911,<sup>68</sup> namely in connection with the aging of the space travellers and the physical laws in the accelerated rocket frame. The term "bolide de Langevin" is also sometimes used in this class of problems.

Langevin's most interesting example in 1911<sup>68</sup> is his charming analysis of the asymmetrical aging due to the difference in motion of twins. A traveller could devote 2 years of his life to be able to visit earth 200 years later. After that accomplishment it would be impossible for him to return into an earlier time to inform of his adventures for any such attempt would only send him further into the future. If the moving observer moves with a constant speed to a star and back such that  $v \approx c$ , or specifically  $1 - v/c = 1/20,000$ , then for a total trip of 2 years, each part taking him a year by his clock, would take 200 years as observed from rest (earth), since<sup>69</sup>  $\gamma = 100$  and the star is 100 light-years distant.

It is amusing to consider Langevin's summary of how the two observers *see* each other's life, not correcting for the time of travel of the light, so that this is distinct from what is obtained by the Lorentz transformation. (Confusion has resulted in the past from inability to realize this distinction.) They can communicate by light-signals or telegraphy in order to try to understand how the asymmetry is possible. During the separation each sees the other flee before the respective emitted signals so that it takes them a longer time to receive the signals emitted in a given time. They see each other live<sup>70</sup> 200 times slower than ordinarily. In the first year the traveller receives news of only somewhat less<sup>71</sup> than the first two earth days after departure; he sees the earth-observers live two days only. Because of the Doppler principle he receives the radiation from earth with 200 times longer wavelength. What he sees as visible light was emitted as extreme ultraviolet (near x-rays). For both sides to receive radio waves the transmitting antenna on earth should be  $1/200$  the length of the traveller's receiving antenna, and the reception

<sup>67</sup> Langevin, 1911 B. p. 42.

<sup>68</sup> Langevin, 1911 A, pp. 47, 50-53. It is interesting to note that he was more careful than many later relativistic authors, in his later stressing that a finite rocket accelerated by an attached "rope" will be subjected to deformations by the rope, because of transmission of acceleration to all parts of the rocket, while gravitation acts upon each rocket-particle equally if it is a uniform field. Only for a point-rocket can acceleration be equivalent to a uniform gravitational field.

<sup>69</sup>  $(1 - \beta^2)^{1/2} = [(1 + \beta)(1 - \beta)]^{1/2} \approx 1/100$  because of  $1 - \beta = 1/20,000$  and  $\beta \approx 1$ .

<sup>70</sup>  $[(1 - \beta)/(1 + \beta)]^{1/2} \approx 1/200$  for the longitudinal Doppler effect so that  $\nu' < \nu$  on leaving but  $\nu' > \nu$  on returning (where  $\nu'$  is the traveller's frequency); life is affected because of the frequency of the heartbeats, etc.—Phys. Sci. Stud. Com. *College Physics* (Boston: Raytheon Educ. Co., 1968), p. 583.

<sup>71</sup>  $365/200 = 1.82$  by Doppler: PSSC, *Coll. Physics*, p. 584.



antenna on earth 200 times longer than his sending antenna. On the return trip converse conditions hold. The observers reciprocally view each other's living 200 times accelerated. The traveller sees 200 years pass on earth and sees as light the waves originally emitted from the earth as extreme infrared. For him to receive radio waves, the earth should after the first two days, and for the remaining 200 years, use a 200 times longer sending antenna than the traveller's one, or 40,000 times longer than that used for the first two days.

To understand the asymmetry one should consider this: the earth needs 200 years to receive the signals from the traveller's first year of travel, and *sees* his life 200 times slowed down. At the end of 200 years the earth receives the message of his encounter with the star at which his return starts. The traveller arrives two days later. The earth *sees* him age 200 times faster than usual during *his* second year of travel. The traveller's acceleration results in the asymmetry. He sees the earth recede and approach for a year each, while the earth sees him, only by his signals, recede for 200 years and approach for two days, a time 40,000 times shorter than that of apparent recession.

Langevin pointed out that enormous practical difficulties would occur in implementing such a program. The work of the earth to launch the traveller and his vehicle, of mass one ton, possibly by rotating it for a year at the end of a catapult, would be about  $400 \times 10^9$  horsepower, equivalent to the combustion of at least 1000 km<sup>3</sup> of oil. To start the return from rest, equally enormous difficulties result. One needs a mechanism to absorb the traveller's kinetic energy and restore it with opposite direction. To stop on earth one must dissipate the kinetic energy gradually without a large change in temperature of the traveller's vehicle. A change in temperature of more than  $10^{16}$  degrees would be equivalent to the above energy. Langevin speculated that in a collision with the earth, the rocket would not leave a hole before stopping inside the earth. Only its passage would leave a slight ionization of the air traversed. For example,  $\alpha$ -particles from radium with a speed of 20,000 km/sec leave no trace in matter except some increase in conductivity. Our rocket has a kinetic energy per unit mass 100,000 times larger and is thus a very penetrating radiation.

#### B. *The Problem of Trains*

Already in 1922 Einstein irritated<sup>72</sup> some Germans by making a visit to Langevin in Paris. His reception there was not<sup>73</sup> always warm, as evidenced for example, by the welcoming words<sup>73</sup> of X. Leon, after referring to Langevin as the "apostle of the new Evangile": "To-day we rejoice in resuming the discussion in the presence of the monster [Einstein] himself; yet a regret constricts our heart." Although Leon humorously welcomed Einstein, the words show the opposition to Einstein by many other Parisians.

<sup>72</sup> Frank, *Einstein*, pp. 194-198.

<sup>73</sup> see *Bull. Soc. Franç. Phil.*, 1922, 17: 92.

Langevin gave a second interesting analysis<sup>74</sup> of the twin paradox (as applied to trains) at another conference at that time in Einstein's presence. Painlevé presented to Einstein the twin paradox as applied to a train moving to the right, passing a station and later returning, its speed always uniform and the changes in motion abrupt. The paradox referred to supposed reciprocal views obtained by inertial station and train observers about aging. Einstein repeated what he already replied to similar objections in 1918<sup>75</sup> by pointing out that in special relativity only the station belongs to an inertial frame, but not the (sometimes) accelerating train. Langevin gave a more complete solution, the next day.

Consider both systems supplied with clocks (lining the train and track), each set of clocks synchronized in its own system. Define  $x$  as the distance from the station in that system;  $x'$  as the distance in the train system from the train center;  $t$  is the time shown by a clock on the track at  $x$ ;  $t'$  is the time of a train-clock at  $x'$ . Let the train-center pass the (small) station at  $t = t' = 0$  for the clocks at  $x = x' = 0$  respectively. For the first part of the trip  $t = \gamma(t' + vx'/c^2)$  and  $x' = 0$  gives  $t = \gamma t'$ , so that the train-center clock is running slower than those it passes on the track; similarly for the return trip, since  $(-v)^2 = v^2$ . Reciprocally, the station clock at  $x = 0$  compared to the passing train clocks runs slower because  $t' = \gamma(t - vx/c^2)$  with  $x = 0$  gives  $t' = \gamma t$ . (Both cases represent time connected intervals.)

The train stops at  $t = T$ , or  $x = vT$  for the center, when its clock reads  $t'_1 = \gamma(T - v^2T/c^2) = T/\gamma$ . For the return-part

$$t'' = \gamma(t + vx/c^2) + K, \quad K = -2\gamma\beta^2T$$

is needed for agreement of  $t''_1$  with  $t'_1$  at  $x = vT$ , since  $\gamma(t + vx/c^2)$  alone would give  $\gamma T(1 + \beta^2) \neq t'_1$ . Upon reversal the train clocks will not mutually agree anymore (without resetting) so that a new synchronization is needed, whence  $t'' \neq t'$  at  $t = T$  except for  $x = vT$ . Thus the center-clock is kept unchanged (for it may not be touched since it will be compared with the station clock upon return) while all other clocks of this very long train will receive an adjustment<sup>76</sup> depending on their position at  $x$ . The returning frame of reference may also be considered as another train moving with  $(-v)$ , whose clocks are shifted by  $K$ . The train clock at the station  $x = 0$  at  $t = T$  showed  $t'_2 = \gamma T$ , but upon resetting  $t'_2 = \gamma T(1 - 2\beta^2)$ . Langevin concluded that the abrupt change can be felt by the shock imparted to the observers; but even if they slept through, they would later find a trace of the

<sup>74</sup> Morand, *La Nature*, 1922, 50: 316-318; see also Stephenson and Kilmister, "Relativity," pp. 43-44. Between Langevin's 1911 and 1922 works one may note the following works on this topic: Laue, *op. cit.*; H. Lorentz, *Das Relativitätsprinzip* (Leipzig: Teubner, 1914), pp. 47-50; *idem.*, *Revue gén. sci.*, 1914, 25: 185-186; and reference 75.

<sup>75</sup> Einstein, *Naturwiss.*, 1918, 6: 697-702; For early comments on this work in *Naturwiss.* see E. Gehrcke, 1919, 7: 147 (criticism) and H. Thirring, 1921, 9: 209 (defense).

<sup>76</sup> Its amount  $a(x) = 2\gamma vx/c^2 - 2\gamma\beta^2T$  I found from

$$\gamma(T + vx/c^2) - 2\gamma\beta^2T = \gamma(T - vx/c^2) + a(x),$$

e.g.  $a = 0$  for  $x = vT$  and  $a = -2\gamma\beta^2T$  for  $x = 0$ .

change since all of the train clocks would be upset, or not in mutual synchronism anymore.

The train-center clock shows  $t_3'' = 2T\gamma + K = 2T/\gamma < 2T$  but the station shows  $2T$  upon the return passing. The center-clock has shown a range  $0 \rightarrow 2T/\gamma$  but the clocks on the track seen to pass by the center showed  $0 \rightarrow 2T$ . But the station also finds its clock to move slower than those seen passing by, for its clock shows  $0 \rightarrow 2T$  while the clocks instantaneously in front of the station give a total elapsed time (not counting the resetting as elapsed time)<sup>77</sup>

$$t_2' + t_3'' - t_2'' = \gamma T + (2T\gamma + K) - (T\gamma + K) = 2T\gamma > 2T.$$

It is instructive to compare this with Einstein's<sup>78</sup> solution of the "paradox" (in 1918) in general relativity by replacing acceleration by gravitational fields. The slowing down (acceleration to the left) of the moving clock in the station system is equivalent to considering the train clock as fixed and under the influence of a gravitational field to the right, while the station system is then moving to the left. The gravitational field will stop the moving landscape originally moving to the left relative to the train. The tower of the station does not topple because it freely falls or floats together with the ground in the gravitational field, while the train is held fixed by external forces. There is a time lag of the station clocks during the uniform motion (no gravity) but during the action of the gravitation it can be shown that this lag is overcompensated thus giving a net lag for the train clocks. Both views, using the station system or the accelerated train system, are equally valid in principle but the latter is less convenient since, as Einstein said: "... [the locomotive-conductor] will object, that he really need not continuously heat and oil *the country*, but rather the locomotive. . . ."<sup>78</sup>

## V. Relativistic Mass and Energy

The Newtonian mass was fundamental or absolute, irreducible to simpler phenomena, and a priori invariable regardless of mechanical, physical or chemical changes in the body. According to Langevin "... the absolute mass is the daughter of absolute time."<sup>79</sup> Relativistic mass on the contrary has a relative significance: it depends on velocity and the chemical and physical internal state (internal energy) and hence on exchange of energy with the environment.

### A. Langevin's Unpublished Work Presented by Others

Langevin's lost contributions on mass-energy and Thomas precession remain in part a mystery. A small puzzle is Einstein's<sup>80</sup> remark in 1912 giving Langevin credit for orally pointing out (to Einstein) that the relation  $\Delta E = \Delta m_0 c^2$  (for inertial

<sup>77</sup>  $2T\gamma$  has a different significance than  $2T/\gamma$ , so that there is no contradiction.

<sup>78</sup> Einstein, *Naturwiss.*, 1918, 6: 701; see also M. Born, *Einstein's Theory of Relativity* (New York: Dover Publ., 1962, revised ed. of 1924 original), pp. 345–346.

<sup>79</sup> Langevin, 1931, p. 286.

<sup>80</sup> Einstein, *Ann. Physik*, 1912, 38: 1062.

mass) must require an equal increment  $\Delta m_g c^2$  (for gravitational mass) for *agreement with experience*, e.g. for equal acceleration in the same gravitational field of bodies of different mass, undergoing radioactive transformations. Although Einstein had this theory in 1907 and June 1911,<sup>81</sup> before their first meeting in October 1911, may one conclude that Langevin influenced Einstein's original work?

At the conference in 1922, according to Morand:

Mr. Langevin received from Einstein the most merited praises. The latter recalled that Mr. Langevin already made himself quite famous . . . by first discovering the inertia of energy.<sup>15</sup>

His work was not published and his personal notes were destroyed in the war (1941).<sup>82</sup> Some evidence remained in notes and statements by his students;<sup>83</sup> E. Bauer, in 1956 spoke of his own discovery (in 1905) of Einstein's publication of Langevin's formula  $E = mc^2$ , but his "testimony" has the unreliable feature of quoting the exact words spoken by Langevin fifty years before. Bauer also explained that Langevin then did not publish his results because of Einstein's article. One may also find some evidence in Langevin's hint of a connection between inertia and radiated energy of an electron, at the end of an article in 1905.<sup>84</sup>

In his unpublished college lectures,<sup>85</sup> starting in 1906, Langevin derived the mass-energy relationship and other dynamical formulae by a simple method, starting from the principle of relativity (relativistic kinematics, e.g. velocity addition) and conservation of energy. Some of his work was fortunately preserved by others. One may summarize from F. Perrin's account.<sup>86</sup>

Langevin made assumptions about energy based on isotropy of space or independence of sign of velocity. The energy  $e_0$  in a system  $S$  will be measured as  $e_0 \phi(u^2)$  in another system  $S'$  moving with speed  $u$  relative to  $S$ , where  $\phi$  is a universal function. Similarly, the kinetic energy and momentum of a particle moving with speed  $v$  in  $S$  are  $T = m_0 f(v^2)$  and  $p = m_0 v g(v^2)$  respectively, where  $m_0$  is the rest mass, and  $f$  and  $g$  are universal functions. The three functions will be determined by simple considerations.

Consider two reference frames (wagons)  $S_1$  and  $S_2$  moving with respect to a "stationary" system  $S$  with equal and opposite velocities ( $-u$ ) and  $u$ , and in each frame  $S_1$  and  $S_2$  there are two moving bodies of unit (rest) mass with velocities,  $(-v)$  and  $v$ . First assume  $v \parallel u$ . With respect to  $S$  by symmetry, the speeds of the two bodies moving in the same sense as their corresponding wagons will have the same speed  $v'$ , while the other pair will have  $v''$ . What is the energy required to stop all

<sup>81</sup> Einstein, *Jahrb. Radioakt.*, 1907, 4: 454-462; *Ann. Physik*, 1911, 35: 898-908.

<sup>82</sup> Staroselskaya, *Lanzheven*, p. 91.

<sup>83</sup> *ibid.*, p. 118. There remains also an impressive notebook on Langevin's relativity classes, taken by Léon Brillouin (1889-1969) in 1911; it is now at the Amer. Inst. Physics collection (whose librarian J. Warnow I thank for courtesy).

<sup>84</sup> Langevin, *Jour. de Phys.*, 1905, 4: reprint in his *Oeuvres*, p. 328.

<sup>85</sup> See also Langevin, 1913, p. 414 and Arzelès, *Dyn. Rel.*, Vol. I, pp. 21-22.

<sup>86</sup> F. Perrin, *Actual. sci. ind.* (1932), no. 41.

four bodies in  $S$ ? There are two possibilities: 1) stop them in  $S$  giving the energy as  $2f(v'^2) + 2f(v''^2)$ , and the total momentum in  $S$  is zero by symmetry; 2) stop the two bodies in  $S_1$ , and the two in  $S_2$  giving  $4f(v^2)$  with respect to  $S_1$  and  $S_2$  or  $4f(v^2)\phi(u^2)$  with respect to  $S$ . There is an additional  $4f(u^2)$  for stopping in  $S$ . Again the total momentum is zero. Conservation of energy demands equality of the two changes:

$$2f(v'^2) + 2f(v''^2) = 4f(v^2)\phi(u^2) + 4f(u^2).$$

If one chooses  $\mathbf{v} \perp \mathbf{u}$ , then all four bodies have the same speed  $v'''$  with respect to  $S$  by symmetry. By a similar reasoning as before one has:

$$4f(v'''^2) = 4f(v^2)\phi(u^2) + 4f(u^2).$$

Now one has two equations, where  $v'$ ,  $v''$ ,  $v'''$  can be replaced in terms of  $u$  and  $v$  using velocity composition formulae. The classical case gives (within a constant factor)

$$f = \frac{1}{2}v^2, \quad \phi = 1, \quad T = \frac{1}{2}m_0v^2$$

Relativity (*via* the velocity addition) gives

$$f = c^2(\gamma - 1), \quad \phi = \gamma, \quad T = m_0c^2(\gamma - 1).$$

Analogously one can obtain  $g = \gamma$  giving  $\mathbf{p} = m_0\gamma\mathbf{v}$ .

To establish the mass-energy connection consider a stationary piece of ice melting (in a fixed system  $S$ ), absorbing an energy  $e_0$  in  $S$ , or  $e = e_0\phi(v^2)$  in  $S'$ , moving with speed  $v$ . The change can be effected in two different ways both giving the same total change measured. Firstly, the ice is fixed in  $S$ , and the energy absorbed is  $e_0$  in  $S$ . Secondly, the ice acquires a speed  $v$  in  $S$  to become at rest in  $S'$ , with an energy change  $m_i f(v^2)$ , with  $m_i$  as the rest mass of ice. Melt it now; the energy change is  $e_0$  in  $S'$  but is  $e_0\phi(v^2)$  with respect to  $S$ . Then stop the water (melted ice) giving in  $S$  a change  $-m_w f(v^2)$ , with  $m_w$  as the rest mass of water, where  $m_i \neq m_w$  is possible because ice is different from water. Both cases must give the same change:

$$e_0 = m_i f(v^2) + e_0\phi(v^2) - m_w f(v^2),$$

or

$$\Delta m_0/e_0 = (m_w - m_i)/e_0 = (\phi - 1)/f.$$

If  $\phi = 1$  (classical case), then  $m_i = m_w$  or upon a change in internal energy there is no change in mass. For  $\phi \neq 1$  a change in internal energy is proportional to a change in mass or  $\Delta m_0 \propto e_0$  because  $\Delta m_0/e_0$  cannot depend on  $v$  of the auxiliary system, or  $(\phi - 1)/f = K$ , a universal constant. Substitution of the relativistic values gives  $K = 1/c^2$  or  $\Delta m_0 = e_0/c^2$ , the plus sign meaning  $m_i < m_w$  because of the heat gain (at the same temperature  $\theta = 0^\circ\text{C}$ ).

### B. Langevin's Derivation of 1913

Langevin gave another valuable mass-energy derivation. In 1913,<sup>87</sup> he treated a similar model to Einstein's 1905 box emitting electromagnetic waves in opposite directions.<sup>88</sup> Consider a rectangular box (whose restframe of reference is  $S'$ ) in motion normal to one of its sides with speed  $v$  to the right, with respect to a "fixed" frame  $S$ . Finite wave-trains are emitted from the box symmetrically to the right and left each with energy  $1/2 \Delta U_0$  in  $S'$ , during a certain time interval, say one second of  $S'$ , so that by conservation of momentum the box stays at rest in  $S'$ . In  $S$  the energy of the wave-trains will be shown to be

$$\Delta U_1 \approx \frac{1}{2} \Delta U_0 (1 + \beta), \quad \Delta U_2 \approx \frac{1}{2} \Delta U_0 (1 - \beta)$$

(neglecting  $\beta^2$  terms so that  $\gamma \approx 1$ ), or momentum

$$\Delta G_1 = \Delta U_1/c, \quad \Delta G_2 = \Delta U_2/c$$

respectively for right and left. Conservation of *total* momentum requires  $\Delta G = \Delta G_1 - \Delta G_2 = \Delta U_0 v/c^2$  to be equal to the change in momentum of the box  $\Delta G \approx v \Delta m_0$  (where again  $\gamma \approx 1$  and  $\Delta m_0$  is the change of rest-mass of the box), so that  $\Delta m_0 \approx \Delta U_0/c^2$ . The case of absorption of energy gives the same final result.

The unusual feature of this derivation is in not using relativity explicitly but only Langevin's own derivation of Lorentz's 1895 transformation of electric fields (to be shown below) and the electromagnetic result  $G = U/c$  essentially obtained by Poincaré (1900).<sup>89</sup> The derivation of  $\Delta U_1$  is as follows: the energy density emitted is  $(1/2 \Delta U_0)/c$  per unit area and time in  $S'$ , or  $\{1/2 \Delta U_0 (1 + \beta)^2\}/c$  in  $S$ , because the electric field is  $E_1 = \gamma E_0 (1 + \beta) \approx E_0 (1 + \beta)$ ; and as the waves "occupy" a length  $\gamma(c - v)$  in unit time (of  $S'$ ), Langevin obtained

$$\Delta U_1 \approx \frac{1}{2} \Delta U_0 (1 + \beta)^2 (c - v)/c \approx \frac{1}{2} \Delta U_0 (1 + 2\beta)(1 - \beta) \approx \frac{1}{2} \Delta U_0 (1 + \beta)$$

to first order in  $\beta$ . Einstein used energy conservation in his derivation in 1905, and (similarly to Langevin) momentum conservation in 1946,<sup>88</sup> both methods being to first order. In 1911,<sup>90</sup> Lorentz also considered our model of the radiating box (without approximations) in detail using both energy and momentum conservation in relativity, but still Langevin's method is different, simpler and more elementary.

It is interesting to note Langevin's derivation of the electric field transformation, for the case of a plane wave travelling to the right, and studied by observers  $S$  and  $S'$  (moving relatively to  $S$  with speed  $v$  along  $x$ ). He did not use Lorentz transformations of fields but used the Lorentz force to first order. Both observers find the wavespeed  $c$ , but  $S$  finds greater field-intensities. For  $S'$  the force per unit

<sup>87</sup> Langevin, 1913, pp. 418–419.

<sup>88</sup> Einstein, *Ann. Phys.*, 1905, **18**: 639–641; see also *Technion J.*, 1946, **5**: 16–17.

<sup>89</sup> H. Poincaré, *Arch. néerl.*, 1900; reprint in *Oeuvres de H. Poincaré* (Paris: Gauth.-Villars, 1954), Vol. 9, p. 476. See also Langevin, 1913, p. 408.

<sup>90</sup> H. Lorentz, *Amst. Versl.*, 1911, **20**: 87–98.

test charge (at rest in  $S$ ) is  $F_0/Q = E_0 + vB_0/c = E_0(1 + \beta)$  because of  $E_0 = B_0$ . In  $S$ , neglecting second-order terms ( $\gamma$ ) in the Lorentz transformation for  $F$ , Langevin had  $F \approx F_0$  or  $E_1 = F/Q = E_0(1 + \beta)$ . For an opposite propagation direction of the wave  $E_2 = E_0(1 - \beta)$  holds since the electric or the magnetic field reverses direction.

### C. Langevin's Results on Potential Energy and Electrons

One should not be surprised that some of Langevin's results of 1913 or before were novel, because the theory evolved slowly throughout 1900–1913; Experimental confirmation was lagging too because of the difficulties of measuring extremely small changes in mass. It is interesting to note that the mass corresponding to potential energy received little attention<sup>91</sup> until the recent studies by Brillouin.<sup>92</sup> Therefore, one is impressed by Langevin's remark of 1904<sup>93</sup> about the non-additivity of individual masses of a group of electrons unless the interelectron distances are relatively enormous, as is true in practice. Namely, as Lorentz noted in 1909,<sup>94</sup> their electric fields overlap, giving a changed total mass. Attention to the problem was given also by Silberstein in 1911, by Fermi and by Whittaker.<sup>95</sup>

A source of difficulty after the turn of the century may have been the use of several definitions of mass and carelessness in distinguishing them (especially upon wrongly using  $T = 1/2 \cdot mv^2$  for high speeds). Again, Langevin reminds us of several aspects of mass, stressing three definitions of it: 1) measure of inertia, or force divided by acceleration, 2) coefficient in linear momentum, or  $p/v$ ; 3) coefficient in kinetic energy.<sup>96</sup> Only for  $v \ll c$  do these definitions coincide with the rest-mass, otherwise all of them are not the same. Langevin also did some work on the problem of early electron models: Abraham (1902) considered a rigid spherical electron, Lorentz had a flattened ellipsoid, Lorentz-contracted in motion, with transverse dimensions unchanged, while Bucherer and Langevin required the contracted electron's volume to be constant.<sup>97</sup> This last model yielded for the transverse mass  $m_{\perp} = m_0(1 - \beta^2)^{-1/3}$ , and longitudinal  $m_{\parallel} = m_0\gamma^{8/3}(1 - \beta^2/3)$ , the latter given only by Abraham in 1908.<sup>98</sup> Langevin only stated the result for  $m_{\perp}$  without showing a derivation, in contrast to Bucherer.

<sup>91</sup> For an example of how this is overlooked in a good elementary textbook, see the error in E. Purcell, *Electricity and Magnetism, Berkeley Phys. Course* (New York: McGraw-Hill, 1965), p. 154.

<sup>92</sup> L. Brillouin, *Proc. Nat. Acad. Sci.* (Wash.), 1965, 53: 475–482, 1280–1284.

<sup>93</sup> Langevin, 1904 B, p. 268.

<sup>94</sup> H. Lorentz, *Theory of Electrons* (New York: Dover Publ., orig. ed. 1909 with notes of 1915), p. 47.

<sup>95</sup> E. Whittaker, *History of the Theories of Aether and Electricity* (New York: Harper Torchbooks, 1953), Vol. II, p. 54.

<sup>96</sup> Langevin, 1913, pp. 397–398, 409–412.

<sup>97</sup> A. Bucherer, *Mathematische Einführung in die Elektronentheorie* (Leipzig: Teubner, 1904), p. 57 ff; Langevin, 1904 B, p. 267.

<sup>98</sup> M. Abraham, *Theorie der Elektrizität* (2nd ed., Leipzig: Teubner, 1908), Vol. II, pp. 197, 399.

It will be recalled that there was an objectionable factor of  $4/3$  in the electromagnetic rest-mass of a spherical electron (in terms of its potential energy  $U_0$ )  $\bar{m}_0 = 4U_0/3c^2 = 4m_0/3$  (where  $m_0 = U_0/c^2$ ), before Fermi in 1922 (and later others) removed it in a basic way.<sup>99</sup> However, using the Poincaré stress, one can argue the factor away, as Langevin did in 1911 (for  $v = 0$ ).<sup>100</sup> He employed Poincaré's and Lorentz's<sup>101</sup> idea of using a potential energy corresponding to the Poincaré stress. Langevin considered only the new energy at *rest*, while Lorentz had done it for *all speeds*.<sup>101</sup> Although Lorentz could have obtained the agreement of the energy equation with relativity, he did not seem to realize it. Laue had a similar treatment to Langevin's in 1911 and 1913, but it was unclear and he did not criticize the  $4/3$ -factor.<sup>102</sup> For  $v \ll c$ , the stress for a spherical electron with surface charge is expressed as a constant pressure  $p = 2\pi\sigma^2$  (where  $\sigma = e/4\pi R^2$  is the surface charge density and  $R$  is the *equilibrium* radius of the electron) or  $p = e^2/8\pi R^4$ . To this corresponds an inner potential energy.

$$\tilde{U}_0 = pV = p4\pi R^3/3 = e^2/6R = U_0/3$$

in terms of the electrostatic energy  $U_0 = e^2/2R$ . Adding them gives the total energy

$$\bar{U}_0 = U_0 + \tilde{U}_0 = 2e^2/3R = 4U_0/3.$$

Comparing with  $\bar{m}_0 = 2e^2/3Rc^2$  one obtains  $\bar{m}_0 = \bar{U}_0/c^2$ , which is remarkable. This assumes the validity of the ordinary electrostatic laws inside and just outside the electron. In summary:

$$\bar{m}_0 = (U_0 + pV)/c^2 = \bar{U}_0/c^2 = 4U_0/3c^2.$$

Langevin also pointed out that this  $\tilde{U}$  makes  $\bar{U}_0$  minimum for the equilibrium radius.<sup>103</sup>

#### D. Applications to Nuclear Physics and Stars

Langevin applied  $\Delta E = c^2\Delta m$  to nuclear physics and the chemical structure of the elements in 1913.<sup>104</sup> According to the speculation of Prout atomic masses of elements are integral multiples of the atomic mass of hydrogen.<sup>105</sup> However, there were deviations from this. Firstly, the effect of isotopes was shown responsible for *large* deviations from integers (such as 35.5 for atomic "weight" of chlorine).

<sup>99</sup> For the topics of electromagnetic mass,  $4/3$ -factor and Fermi see A. Gamba, *Am. J. Phys.*, 1967, 35: 86–88.

<sup>100</sup> Langevin, 1913, pp. 413–414; also in Brillouin's 1911 notebook.

<sup>101</sup> H. Lorentz, *Theory of Electrons*, pp. 213–214.

<sup>102</sup> M. von Laue, *Relativitätssprinzip* (Braunschweig: Vieweg) 1st ed. of 1911, pp. 164–167; 2nd ed. of 1913, pp. 198–199.

<sup>103</sup>  $\bar{U}_0 = e^2/2R + 4\pi R^3 p/3$  is minimum when  $R$  assumes the equilibrium value in

$$\frac{\partial \bar{U}_0}{\partial R} = -e^2/2R^2 + 4\pi p R^2 = 0.$$

Then follows  $p = e^2/8\pi R^4 = \text{const.}$

<sup>104</sup> Langevin, 1913, pp. 422–424.

<sup>105</sup> See Whittaker, *History . . .*, Vol. I, p. 361.



Secondly, there were *small* discrepancies, which fact was an obstacle to the theory of formation of complex atoms from simpler ones since that would violate the classical conservation of mass. Langevin accounted for the small nuclear mass-defect  $\Delta m$  in terms of the energy change  $\Delta E = c^2 \Delta m$  as liberated energy. For example, the atomic "weight" of oxygen is 15.87 (based on  $H = 1$ ;  $H = 1.008$  gives  $O = 16$ ), while that of 16 hydrogen atoms is 16. The difference is to be interpreted as the energy liberated in the "formation" of oxygen, since mass-energy is conserved. Langevin did not mention isotopes in 1913. These were already known to exist for heavy elements but for the lighter ones they were discovered just at that time by J. J. Thomson and F. Aston, and named by Soddy.<sup>106</sup> Previous work of interest had been done by Einstein in 1907,<sup>107</sup> who considered the energy liberated in radioactive decay to be given by  $E/c^2 = M - \sum m_i$  (where  $M$  is the initial rest mass of the decaying atom and  $m_i$  are the masses of the decay products). Comstock,<sup>108</sup> in 1908 had a similar theory to Langevin's but used the wrong relationship  $\Delta m = 4\Delta E/3c^2$ . Swinne anticipated Langevin's results in 1913.<sup>109</sup>

In this connection, it is interesting to note that, according to Sambursky, in 1871 Mendeleev remarkably anticipated the mass-energy connection in atomic phenomena:

... there is no reason to suppose that  $n$  parts of the weight of an element or  $n$  atoms will yield the same  $n$  parts after transmutation into an atom of another element, *i.e.* that the atom of the second element will be  $n$  times heavier than the first. One can regard the law of conservation of weight [*i.e.* mass] as a special case of the law of conservation of force [*i.e.* energy] or of movement. Surely weight depends on a special kind of movement of matter, and there is no reason to deny the possibility of a transmutation of these movements into chemical energy or some other form of movement during the formation of elementary atoms. . . . Thus in case a known element would be decomposed or a new one would be formed, these phenomena could well be accompanied by a decrease or increase in weight. In this way one also could explain to a certain extent the difference in chemical energy of various elements.<sup>110</sup>

It was characteristic of Langevin to adopt fruitful new ideas however revolutionary they were and to develop them further.<sup>111</sup> He expounded J. Perrin's and Eddington's ideas of 1920 that solar energy is furnished by atomic fusion.<sup>112</sup> This would release enormous energies such that the age of the sun could be billions of years. Chemical and gravitational mechanisms had been shown to be insufficient.

<sup>106</sup> *ibid.* Vol. II, pp. 12–13.

<sup>107</sup> Einstein, *Jahrb. Radioakt.*, 1907, 4: 442–443.

<sup>108</sup> D. Comstock, *Phil. Mag.*, 1908, 15: 1–21.

<sup>109</sup> R. Swinne, *Phys. Zeitschr.*, 1913, 14: 145–147.

<sup>110</sup> See R. Sambursky, *Isis*, 1969, 60: 104–106.

<sup>111</sup> See Langevin's articles starting 1928: references 37–39.

<sup>112</sup> J. Perrin, *Revue du Mois*, 10 fév. 1920, 21: 113–166; A. Eddington, *Brit. Assoc. Adv. Sci. Repts.*, 1920, 88: 34–49.

The sun's mass would then steadily decrease by the loss of energy, as Comstock concluded in 1908.<sup>108</sup> Another interesting idea adopted by Langevin was the mutual annihilation of positive and negative charges, such as the proton and electron; Eddington (1919) considered this for electrons and nuclei, in connection with the energy source of stars.<sup>113</sup> However, Larmor<sup>113</sup> in 1897 in his model of ether had remarkably thought of annihilation of positive and negative "electrons," long before Dirac's positron of 1930.

In 1913 Langevin anticipated the enormous atomic energy release that he unfortunately witnessed before his death in 1946. He wrote in a section entitled "Matter, reservoir of energy".

Should one consider that all inertia of matter has no other origin? . . . To all inertia would correspond the presence in the system . . . of an energy whose liberation would correspond to complete destruction of the material structure.

Without judging now whether we shall some day acquire this destructive power and exhaust the reserves of energy present in matter, we can, . . . evaluate the importance and enormity of such reserves. Each gram of matter, whatever be its nature, would correspond to the presence of an internal energy equal to  $9 \times 10^{30}$  ergs, that is equivalent to a heat that would be furnished by the combustion of  $3 \times 10^9$  g or 3 million kg of oil.<sup>104</sup>

In 1931 and 1933 he said:

. . . this complete destruction of matter, . . . would constitute an explosive phenomenon, and the man who released it, then a sorcerer's apprentice, would cause a worse catastrophe than all we can imagine.

Fortunately perhaps for the security of our species no Prometheus yet has come to teach men how one can light the scintillating fire of nuclear reactions.<sup>114</sup>

Planck in 1907 also spoke of the enormous reservoir of inner energy,<sup>115</sup> Eddington in 1920 considered sub-atomic energy, abundant in matter, and of the dream of man's using it for the benefit or suicide of the human race.<sup>115</sup> Even in 1899, Heaviside had written:

All known disturbances are conveyed either electromagnetically or gravitationally . . . Assuming then that all disturbances are conveyed at finite speed, it follows instantaneously that the destruction of this wicked world may come at any moment without any warning. There is no possibility of foretelling this calamity (or blessing possibly), because the cause thereof cannot give us any information till it arrives, when it will be too late to take precautions against destruction . . . As the universe is boundless one way towards the great, so it is equally boundless the other way towards the small, and important events

<sup>113</sup> A. Eddington, *Observatory*, 1919, 42: 375; see also J. Larmor, *Phil. Trans.*, 1897, 190: 209-212.

<sup>114</sup> Langevin, 1931, p. 288; 1933, p. 144.

<sup>115</sup> M. Planck, *Ann. Physik*, 1908, 26: 30; see also Eddington, above 1920 article, pp. 45-46.

may arise from what is going on inside of atoms, and again in the inside of electrons. There is no energetic difficulty. Large amounts of energy may be condensed by reason of great forces at small distances. How electrons are made has not yet been discovered. From the atom to the electron is a great step, but it is not finality.<sup>116</sup>

## VI. Langevin's Wit

J'ai voulu présenter des lumières et des ombres pour  
faire un tableau un peu vivant de notre situation.  
Heureusement, les lumières sont riches et les ombres  
sont pleines de promesses.

LANGEVIN (1933)

Langevin's wit and poetic use of language were incomparable (even in English translation). We may consider him as

... a good guide toward the high peaks recently discovered and the great horizons on which, here and there, floats still a little morning fog, but where our avant-garde has already explored marvellous lands.<sup>117</sup>

His views on cosmology:

... Einstein... opened to us ... a new window to eternity.

Thus results a calming of our apprehension that we experience before the infinite, since we feel enveloped in a finite universe, ... which, ... expands constantly as if to make more space to the human spirit.

We need not fear a housing crisis in such a space.<sup>118</sup>

Langevin reminds us that even "games of the spirit" such as non-Euclidean geometry may have possible later applications; thus "... one sterilizes scientific research by prematurely obliging it to occupy itself with material interests."<sup>119</sup>

His views on the crisis of classical physics: "Einstein broke all those idols of absolute time, space and ether. ..."<sup>120</sup> The difficulties originated in part because physicists had "absorbed the Newtonian virus" or were "contaminated by habit."<sup>121</sup> The crisis was overcome by

... [the] beneficial storm, by which physics became rejuvenated. ... , breaking out around 1910 from the dark clouds that [one] ... saw accumulate since 1900 on the horizon of the pure sky of the triumphant electromagnetic theory.<sup>122</sup>

Langevin linked in a very original way this struggle to the conflict between mechanics and electromagnetism, or action-at-a-distance against the gradual propagation of

<sup>116</sup> O. Heaviside, *Electromag. Theory*, Vol. III, p. 519 (1912).

<sup>117</sup> Langevin, "Préface," in Bauer, *Relativité*.

<sup>118</sup> Langevin, respectively: 1922, p. 22; 1931, p. 297; 1922, p. 4.

<sup>119</sup> Langevin, 1922, p. 4.

<sup>120</sup> Langevin, 1933, p. 148.

<sup>121</sup> Langevin, 1922, pp. 7-8.

<sup>122</sup> Langevin, 1933, p. 150; see also Langevin, in *L'Orientation actuelle des sciences*, Confér. École norm. supér. (Paris, Alcan, 1930), p. 46.

effects, but more basically to "the so-called common sense, *i.e.* the necessarily superficial and limited experience of our ancestors."<sup>120</sup>

By a natural and legitimate tendency, the spirit seeks to explain the unknown by the known and to utilize, in new domains, the means of representation that succeeded in the limited framework of past experience, the notions to which it is accustomed by habit. Readily confusing the familiar with the simple, it tends to attribute a universal and absolute value to results verified only in the limited region that it has already recognized, cultivated and sowed. And a crisis results each time when . . . experience deceives the imprudently conceived hope and requires an adaption of ancient ideas to the representation of a new domain. Thus continues the life of the spirit in its evolution toward greater and more comprehensive syntheses.<sup>122</sup>

Langevin's following two remarks about early relativity sound more familiar. Kuznetsov<sup>123</sup> reported that Langevin thought once that only twenty people understood relativity theory, while Einstein denied having said this (since he thought any physicist can easily understand it, as did his students in Berlin). G. Bachelard wrote: "Tensor calculus, Paul Langevin liked to say, knows relativity better than the relativist himself."<sup>124</sup>

### Acknowledgement

I am grateful to Professor A. Woodruff for his patience and his remarkable corrections and clarifications, in particular of the twin problem.

### Appendix

#### A. Trouton-Noble Experiment

In this experiment, which seeks to demonstrate an effect of order  $v^2/c^2$  of the earth's motion with respect to the ether, a plane capacitor fixed on earth, with plates oriented at an angle with respect to the motion of the planet, should turn, as was supposed around the turn of the century. Namely, there would be a magnetic field in the frame of the ether, associated with the moving charged plates and tending to orient the plates parallel to the velocity of the earth. But the experiment (performed in 1902-3) shows no turning tendency in agreement with relativity for regarded from the earth reference frame there is no magnetic interaction.

As pointed out by Butler<sup>29</sup> one can attack this problem either by considering forces or energy and the latter method had not before him given a satisfactory explanation of the null-result for it did *not* show the field energy in the capacitor to be equal for each orientation, thereby giving a turning tendency, contrary to observations. However, Butler overlooked the treatment of Langevin,<sup>20</sup> who by using an energy method first showed there is no turning tendency. G. FitzGerald, F.

<sup>123</sup> B. Kuznetsov, *Einstein* (Moscow: Izdatelstvo "Nauka," 1967), p. 262.

<sup>124</sup> G. Bachelard, in P. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (New York: Harper Torchbooks, 1959 reprint of 1949 ed.), p. 578.

Trouton and H. Lorentz<sup>125</sup> all predicted a turning tendency, while Larmor<sup>126</sup> seemed to favour no turning, if there is a Lorentz contraction, but his work is open to the above objection about energy.

Again Langevin had a simple new method for dealing with the problem—an action principle in analogy with Hamilton's principle in mechanics. The electromagnetic system evolves between two configurations as determined by  $\delta \int_{t_0}^{t_1} (U_e - U_m) dt = 0$ , where  $U_e$  and  $U_m$  are the electric and magnetic energies, and  $L = U_e - U_m$  is the Lagrangian, time independent and an extremum for equilibrium. By considering any charged system, one can show using the Lorentz contraction that the Lagrangian is<sup>126</sup>  $L = L_0(1 - \beta^2)^{1/2}$  in the frame  $S$  where the system is moving, while  $L_0$  is for the rest-frame  $S'$ . This  $L$  is independent of orientation (angle); hence, there is no turning moment. Namely, the difference of energy densities  $u_e - u_m$  is invariant,  $dV' = \gamma dV$  for volume, and  $L = \int (u_e - u_m) dV$ . Butler obtained  $U = \gamma U_0$  for the total energy, which is also independent of orientation. Langevin attributed the null-effect to the compensating effect of the Lorentz contraction.

### B. The Sagnac Experiment

This experiment was performed in 1913 to demonstrate the reality of the ether, but instead it showed that light propagates with a speed independent of the motion of its source.<sup>127</sup> Sagnac wanted to contradict special relativity, but failed since this theory admits the absolute character of rotations; moreover, Langevin explained the experiment by general relativity. As both the classical and relativistic explanations agree, this experiment cannot be used as a test of relativity. It is analogous to the Foucault pendulum, demonstrating the effects of rotation.

Two rays of light from the same source interfere after having traversed a polygonal circuit in opposite directions by reflection from suitably spaced mirrors fixed to a platform. Rotation of the platform produces a displacement of the interference fringes depending on  $R\omega/c$  to first order. (The source and observer rotate with the platform too.)

The classical explanation, for a circular circuit (with many mirrors) for simplicity, considers the speed of light in the rest-frame of the platform  $S'$  as  $c \pm \omega R$  so that

$$\begin{aligned} t &= \frac{2\pi R}{c - R\omega} - \frac{2\pi R}{c + R\omega} \approx 2\pi R[(1 + R\omega/c) - (1 - R\omega/c)] \\ &= 4\pi R^2\omega/c^2 = 4A\omega/c^2 \end{aligned}$$

with  $A = \pi R^2$  to first order in  $\omega$ . In 1921, Langevin (using general relativity) worked

<sup>125</sup> J. Larmor, 1902 note, in G. FitzGerald, *Scientific Writings* (Dublin: Hodges, Figgis & Co., 1902), pp. 566–569.

<sup>126</sup> Abraham obtained the Lagrangian already in March 1905 in his 1st ed. of *Theor. d. Elektr.*, as on p. 189 of the 1908 ed.

<sup>127</sup> A. Metz, *Jour. de Phys.*, 1952, 13: 231; see also d'Abro, *Evol. Sci. Thought*, p. 154.

with

$$ds^2 = c^2 dt^2 - 4\omega dA dt - dl^2,$$

which is valid to first order, where  $dA = 1/2 \cdot R^2 d\theta = 1/2 \cdot (xdy - ydx)$  is the area of a triangle with sides  $R$  and  $Rd\theta$ , the element of length along the ray. Light propagates according to  $ds = 0$ , and solving the resulting quadratic equation in  $dt$  gives to first order  $dt_1 = dl/c + 2\omega dA/c^2$  for one ray and upon integration  $t_1 = l/c + 2A\omega/c^2$ ,  $A$  being the area inside the circuit. For the opposite ray  $dA$  changes sign or  $t_2 = l/c - 2A\omega/c^2$ . Then  $\Delta t = t_1 - t_2 = 4\omega A/c^2$  agrees with Sagnac's and Laue's classical explanations. This experiment measures the influence of Einstein's gravitational potentials  $g_{24} = -2\omega r^2$  (or in rectangular coordinates  $g_{14} = 2\omega y$  and  $g_{24} = -2\omega x$ ) on the motion of light. In 1937 Langevin also obtained the result from  $d\tau = dt - \omega r^2 d\theta/c^2$  giving

$$\tau_1 = \oint d\tau_1 = t_1 - \frac{\omega}{c^2} \oint r^2 d\theta = t_1 - 2A\omega/c^2 \quad \text{and} \quad \tau_2 = t_2 + 2A\omega/c^2$$

for each ray respectively. Then  $\tau_1 = \tau_2$  implies  $\Delta t = 4\omega A/c^2$  as before.

#### Note Added

Additional important works relevant to Langevin were found in Paris (and London) but they did not alter any conclusions of this article. The work P. Langevin, *Notice sur les travaux de P. Langevin* (Paris: Société génér. d'imprimerie et d'édition, 1934) is Langevin's own résumé of his scientific career (apparently written in connection with admission to an academy), containing his extensive bibliography and sections about relativity (pp. 18–19, 49–53, 64–70, 78–83). My thanks to the Bibliothèque d'histoire des sciences, of the Centre international de synthèse, where I found this book. Langevin's selected works are also in P. Lanzheven, *Izbranye Proizvedeniya* (Moscow: Izdat. inostranoi liter., 1949), including an article about Langevin by A. Maksimov (pp. 5–35). Material about relativity is contained in P. Biquard, *Langevin* (Paris: Éditions Seghers, 1969), pp. 44–54, 65, 129; where a Rumanian biography is mentioned: S. Ghimesan, *P. Langevin* (Bucarest: Éditions de la Jeunesse, 1964). Langevin's work on energy and mass was adapted with slight modifications also by G. Allard, in *L'Énergie dans la nature et dans la vie*, 1946 conference (Paris: Presses Univ. de France, 1949), pp. 103–130 and by P. Soleillet and N. Arpiarian, *Éléments de la théorie de la relativité restreinte*, Cours de Sorbonne (Paris: Centre de Docum. Univ., n. d.), pp. 51–56; both are mentioned by Arzelis (my Ref. 1). Other works: O. Starosselskaya-Nikitina, "La Contribution de P. Langevin à la Théorie Relativiste et sa portée historique," *Actes du 8<sup>e</sup> Congrès Internat. d'Hist. des Sciences* (Firenze, 3–9 Sept. 1956), pp. 178–182; J. Nicolle, *La Science au service de l'émancipation de l'homme* (Alger: Éds. Liberté, 1947), in particular p. 20 (see my Ref. 11); R. Lucas, "L'Oeuvre scientifique de P. Langevin," *Cahiers Rationalistes*, nr. 135, Nov.–Dec. 1953, 14–19. There is also my Ph.D. thesis: *A History of Relativity: The Role of Henri Poincaré and Paul Langevin* (Yeshiva University, New York, Sept. 1970).