Thomas HAWKINS: *Lebesgue's Theory of Integration, Its Origins and Development*. University of Wisconsin Press, 1970, 227 pp.

This book deals with a history of the theory of integration, covering the period from about the middle of the 18th century (the time of Euler and d'Alembert) to the early part of the 20th century (the time of Lebesgue and his contemporaries). The emphasis, however, is on the half century or so following Riemann, and Dr. Hawkins devotes almost all of his pages to tracing the evolution of the theory of integration from its early development under Riemann to the (classically) decisive synthesis achieved by Lebesgue.

The book consists of the following chapters:

In addition to these, there is a section headed "Reference Material", which contains, among other things, an appendix, "Dini's Theorem on the Differentiability of Continuous Functions", and a bibliography, in which are listed more than 300 books and articles.

On glancing at this table of contents, one might suspect that, despite the title of the book, only a few pages are assigned to Lebesgue's works. Indeed, it is not until Chapter 5 that Lebesgue's theory is taken up as the main subject. However, the author comments on this matter in his preface as follows:

> Although a relatively small amount of space is devoted to actually discussing Lebesgue's work, it represents the focal point of the entire book. My objective has been to place Lebesgue's early work on integration theory (1901–10) within its proper historical context by relating it, on the one hand, to the developments during the nineteenth century that motivated it and gave it significance and, on the other hand, to the contributions made in this field by his contemporaries. It is my hope that this volume will engender an appreciation for Lebesgue's genius balanced by an appreciation for the manner in which his work builds upon that of many other mathematicians.

We believe that, since this study is based on the detailed investigation of original books and papers, Hawkins' "hope" has, to a large extent, been realized.

Regarding the basic idea of this book, a further point is worthy of remark: that the author's insight into this period of history seems to be influenced by Bourbaki's historical idea found in the first half of his "*Note historique (de l'intégration)*". (This "*Note*" is compiled in his "*Eléments d'histoire des mathématiques*". The first edition of this work, that of 1960, is listed in the bibliography of Hawkins' book.) Although we can not claim definitely that Hawkins' treatment is based on Bourbaki's ideas, we believe that we can identify the latter's influence. This remark is not, of course, intended to detract from the value of Hawkins' book. On the contrary, we are fully aware of the fact that it would be a most difficult and most valuable enterprise to compose a more excellent history than Bourbaki's, in whatever field of mathematics and, in our opinion, Hawkins' work deserves to be called, to a certain degree, an elaboration of Bourbaki's history.

A detailed plan of this history is set out in the "Introduction", and is worked out, step by step, during the course of the following chapters. Firstly, "(1) a fully developed measure-theoretic point of view and (2) a number of theoretical 'problems' that had been discovered within the context of Riemann's definition of the integral" are identified as pre-Lebesgue contributions; then Lebesgue's theory is depicted not only as the measure-theoretic generalization of the integral but also as "an analytical tool capable of dealing with—and to a large extent overcoming—the unsolved problems that had arisen in connection with the old theory of integration". These were problems concerning such matters as the trigonometric series, the "Fundamental Theorem II (*viz.*, the assertion that $\int_a^b f'(x)dx = f(b) - f(a)$)", the length of the curve, the identity of double integrals and iterated integrals. Many important notions and theories in Lebesgue's theory are discussed in connection with these problems. Since the scope of this history is limited to the classical theory, Lebesgue's theory of integration is viewed as the final stage of development, and the "Epilogue" is devoted to a discussion of the Lebesgue-Stieltjes integral, which opened various ways leading this classical theory into more modern fields of research.

Generally speaking, it is the technical aspects of his subject that Hawkins handles most successfully, but the remarks less technical in character which he occasionally inserts between technical details sometimes throw light upon the work as a whole. In particular, the following example, taken from the "Introduction", reveals an important aspect of the author's general historical insight:

> Lebesgue's work on the Fundamental Theorem II and on the theory of curve rectification played an important role in his discovery that a continuous function of bounded variation possesses a finite derivative except possibly on a set of Lebesgue measure zero. This theorem gains in significance when viewed against the background of the century-long discussion of the

differentiability properties of continuous functions. During roughly the first half of the nineteenth century, it was generally thought that continuous functions are differentiable at "most" points, although continuous functions were frequently assumed to be "piecewise" monotonic. (Thus, differentiability and monotonicity were linked together, albeit tenuously.) By the end of the century this view was discredited, and no less a mathematician than Weierstrass felt there must exist continuous monotonic functions that are nowhere differentiable. Thus, in a sense, Lebesgue's theorem substantiated the intuitions of an earlier generation of mathematicians.

We consider such a combination of technical detail and general comment is useful for students both of the history of mathematics and of the pure mathematics. It is one of the strong points of Hawkins' work.

Nevertheless, as a special comment, we must express our regret that the author does not touch upon the concept of non-measurable L. It is well known that on this concept depend important issues such as the axiom of choice, the descriptive set theory, the measure-problem, or, more generally, many problems on the (so-called) foundations of mathematics. It is true that this subject is so difficult and so profound that its treatment would require another book and that, therefore, we must appreciate the way in which Dr. Hawkins, without attempting such a formidable task, has drawn an excellent picture of the theoretical world. Nevertheless, we can not help thinking that he might have made at least passing mention of such an important subject. We also believe that the same criticism apply to Bourbaki's "*Note historique (de l'intégration)*", at least if we view it as being separate from the main mathematical text of "*Théorie d'Intégration*", where these issues in question are not mentioned.

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News

Annual Meeting

The History of Science Society of Japan met for its 18th annual meeting on May 2 and 3, 1971, at Kanda Campus of Senshu University, Tokyo. The following are the papers and the symposium presented on that occasion.

May 2

Women Scientists in Japan and in other Countries — Teruko SEKINE, Aiko YAMASHITA, & Akashiko YOSHIMURA
Mathematical Books in the early Edo Period — Kazuo SHIMODAIRA
Technical Terms of Physics in the early Meiji Period — Manpei HASHIMOTO
H. Nagaoka's Spectroscopy (1908–23)—Social Aspects— — Eri YAGI
Specialization of Chemical Profession in the Meiji Period — Kunika SUGAWARA
On Yoan Udagawa — Tatsumasa DOKE
Saburo Utsunomiya and Kojunsha — Minoru TANAKA
Cytology in Japan—"Protoplasmics" and Gihei Yamaha— — Zenji SUZUKI
History of the River Exploitation — Yoshinori KANEZEKI
A History of "Vital Force" — Tatsumasa DOKE
The Theory of Evolution in the Textbooks of Biology in Japan — Ichio MORI & Masakazu NISHIKAWA
Stages of the World History in relation to the Technical Development — HSIEH Shin-Hui
Development of Automobile Industry in Japan — Hiroo KATO
On the Methods of the History of Physics — Shigeki MATSUO
How to Teach History of Science and Technology — Masao IWAKI
Computor-aided Indexing of *Nihon Kagaku-Gijutsu-Shi Taikei* and a Thesaurus for Historical Documents — Tetsuo TOMITA & Kazutoshi HATTORI

May 3

On Aristotle's Theory of Nutrition of Plants — Kazuo MAFUNE
"Proportionalia" in the Medieval Mechanics — Tsuyoshi OGAWA & Isao OHAMI
Kepler's Process of Conceiving his Harmonic Law — Fukutaro SHIMAMURA
Possible Influences of Huygens' Theory of Impact on Newton's *Principia* — Masahiko YOKOYAMA
A History of Probability: D'Alembert and Condorcet — Eizo YAMAZAKI
The Acceptance of the Abbe Theory of 1873, with reference to Optical Information Theory — Keiichi TSUNEISHI
Planck's Theory of Quantization of the Phase Space — Sigeko NISIO
On van den Broek's Hypothesis — Tetu HIROSIGE

Symposium: Reappraisal of Nineteenth-Century Science

A. Reports

The Making of "Academism" — Shigeru NAKAYAMA

Nineteenth-Century Mathematics — Tamotsu MURATA

Nineteenth-Century Medicine — Yonezo NAKAGAWA

B. Comments — Hajime KASHIWAGI, Zenji SUZUKI, & Kenzo SAKAMOTO

C. Discussion — Tetsuo TSUJI & Kunio GOTO (Chairmen)

New Monographs

The History of Science Society of Japan has recently published the following two monographs:

Martin Levey, Chemical Aspects of Medieval Arabic Minting.

Martin Levey and Mohammad Yadegari, Abū Kāmil's "On the Pentagon and Decagon."

These monographs are supplements to the *Japanese Studies in the History of Science* and are obtainable through JAPAN PUBLICATIONS TRADING CO., LTD., P. O. Box 5030 Tokyo International, Tokyo, Japan.