RESPONSE CHARACTERISTICS OF ELASTIC JOINT ROBOTS DRIVEN BY VARIOUS TYPES OF CONTROLLERS AGAINST EXTERNAL DISTURBANCES

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ABSTRACT

The purpose of this paper is to investigate behaviors of elastic joint robots driven by various types of controllers against external disturbances applied to them. The elasticity is introduced intentionally by installing elastic devises in their transmission. We consider about four types of controllers, commonly used to control elastic joint robots. We investigate the role of the compliance introduced by the mechanisms and the controllers and show numerical simulations.

1 INTRODUCTION

Since active force control strategies, e.g. Hybrid Position/ Force Control by Reibert and Craig[11] and Impedance Control by Hogan[3], have been developed, most of force control strategies have aimed to control contact force actively.

On the other hand, passive compliance devices have been developed to control the contact force passively[13]. Among them Remote Center Compliance (RCC) is the most successful example to implement the mechanical impedance, but it provides the correct impedance only for predetermined peg-in-hole insertion task. In the last decade many mechanical elastic devices have been developed for the advantage of force control tasks and are applied to the humanoid robots.

Laurin-Kovitz *et al.*[7] have proposed Programmable Passive Impedance (PPI) to control the impedance of robots by incorporating programmable mechanical elements into the robots' drive system. Hyodo and Kobayashi[4] have developed Non-linear Spring Tensioner (NST) to control the mechanical stiffness of tendon-driven mechanisms and realized wide range of stiffness adjust-ability. They controlled the robot using a tensile force feedback controller. Pratt *et al.*[10] have developed Series Elastic Actuator (SEA) to realize greater shock tolerance, lower reflected inertia, more accurate and stable force control, and so on. SEA has a motor, a gear train and a spring arranged serially. Morita and Sugano[8] have developed Mechanical Impedance Adjuster (MIA). MIA has a leaf spring and a Pseudo-damper by using a brake and can adjust the spring constant and the attenuation constant. Okada and Nakamura[9] have developed Cybernetic Shoulder (CS) to imitate the kinematic mechanisms of human shoulders. CS has a mechanical viscoelastic link in the closed link chains of the gimbal mechanism. Koganezawa *at al.*[6] has developed Non-Linear Elastic Mechanism (NLEM) to control mechanical stiffness of the tendon-driven mechanisms.

Each mechanism was well-investigated about its mechanical performance and its positioning ability, but merely investigated about the effect of external forces on the whole control systems.

The purpose of this paper is to investigate behaviors of elastic joint robots driven by various types of controllers against external disturbances applied to them. We introduce about four types of controllers; Computed Torque Method, Non-linear Feedback Method with an Exact Linearization, Adaptive Control Method and a PD control Method base on motor angles. They are commonly used to control elastic joint robots. We investigate the role of the compliance introduced by the mechanisms and the controllers and show some numerical simulations.

2 ELASTIC JOINT ROBOTS

2.1 CLASSIFICATION OF ELASTIC JOINT ROBOTS

Table 1 shows the controlled variables and the elasitcity of the prescribed elastic devices. The elasticity is classified into three types; non-linear, linear, and lin-

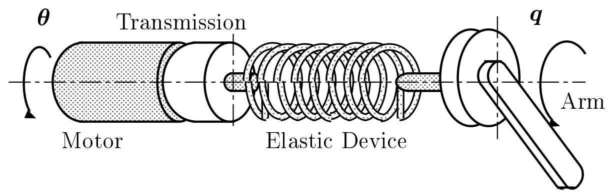

Figure 1: An Elastic Joint Manipulator

Table 1: Elastic performance and control variables

	Control variables	
Elasticity	motor angles	joint angles
non-linear	PPI (Kovitz et al., 1991) NLEM (Koganezawa, 2000)	NST (Hyodo et al., 1993) CS (Okada, 1998)
linear	RCC (Whitney, 1981)	PaCMMA (Morrell, 1998) SEA(Pratt, 1996)
linear, but variable	MIA (Morita et al., 1996)	

ear but adjustable. The controlled variables are motor angles and joint angles. And the class of drive systems are tendon-driven systems or direct motor-driven systems.

2.2 DYNAMICS OF ELSTIC JOINT ROBOTS

Formulation of the elastic joint manipulators requires us to consider the coupling terms between the motors and the joints strictly. But we can elminate the coupling terms when the gear ratio is sufficiently large[12]. In this paper we adopt the assumption because such transmissions are usually located between the motors and the elastic devices. So the equations of motion of the elastic joint robots can be expressed as:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}_1(\boldsymbol{q})^T\boldsymbol{f} = \boldsymbol{f}_{\text{ext}}, \quad (1)$$

$$\boldsymbol{M}_a\ddot{\boldsymbol{\theta}} + \boldsymbol{B}_a\dot{\boldsymbol{\theta}} + \boldsymbol{J}_2^T\boldsymbol{f} = \boldsymbol{\tau}_a. \quad (2)$$

where $\boldsymbol{q}$ and $\boldsymbol{\theta}$ are the joint angle vector, the motor angle vector respectively. $\boldsymbol{M}$ are the inetia matrix of the arm, $\boldsymbol{C}$ the coefficient matrix of the Coliolis and centrfugal force, $\boldsymbol{g}$ the gravitational force vector, $\boldsymbol{J}_1$ the Jacobian from the joint space to the elastic deformation space, $\boldsymbol{J}_2$ the Jacobian from the motor space to the elastic deformation space, $\boldsymbol{f}$ the elastic force vector, $\boldsymbol{\tau}_a$ the drive force vector and $\boldsymbol{f}_{\text{ext}}$ the external force vector.

3 CONTROLLERS FOR THE ELSTIC JOINT ROBOTS

In this section we describe four types of controllers for the elastic joint robots; Computed Torque Method (CTM), Nonlinear control method with Feedback Linearization (NFL), Adaptive Control Method (ACM), and PD control Method base on motor angles (PDM). They are often used in positioning control of varous elastic joint robots. In this section we assume that $\boldsymbol{f}_{\text{ext}}$ is the zero vector.

3.1 THE STRUCTURE OF CTM

Figure 2 shows a block diagram of CTM. The controller has an inner controller to compensate the nonlinearity and the elasticity. The inner controller is designed so as to converge much faster than the outer controller does. The outer controller calcurates only a desired joint torque. Then eq.(1) can be rewitten as follow:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}, \quad (3)$$

where $\boldsymbol{\tau}$ is the joint torque vector. The inputs are given as follows:

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{M}(\boldsymbol{q})\boldsymbol{v}, \\ \boldsymbol{v} &= \ddot{\boldsymbol{q}}_d + \boldsymbol{K}_v(\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_d) + \boldsymbol{K}_p(\boldsymbol{q} - \boldsymbol{q}_d), \end{aligned} \quad (4)$$

where $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are the proportional and derivative feedback gain matrices, respectively, and $\boldsymbol{q}_d$ is the desired joint angle vector. This controller guarantees convergence to the desired trajectories.

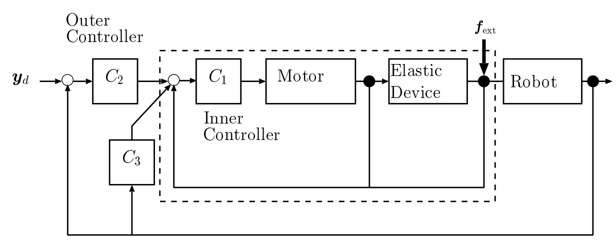

Figure 2: Comupted Torque Methods

3.2 THE STRUCTURE OF NFL

De Luca[2] has showed that dynamic feedback is necessary to linearize the control systems of the elastic joint robots strictly. But in this case a static feedback is enough to linearize the system because of the assumption of neglecting the coupling terms. **Figure 3** shows the block diagram of the controller. Eqs. (1) and (2) can be rewritten as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_x(\boldsymbol{x}) + \boldsymbol{G}_x(\boldsymbol{x})\boldsymbol{\tau}_a, \quad (5)$$

where $\boldsymbol{x}^T = (\boldsymbol{q}^T, \boldsymbol{\theta}^T, \dot{\boldsymbol{q}}^T, \dot{\boldsymbol{\theta}}^T)$. If we define transformation

$$\boldsymbol{z} = \boldsymbol{p}(\boldsymbol{x}) \tag{6}$$

and control input

$$\boldsymbol{\tau}_a = \boldsymbol{\alpha}(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x})\boldsymbol{v}, \tag{7}$$

then we can obtain the following linearized system.

$$\dot{\boldsymbol{z}} = \boldsymbol{A}_z \boldsymbol{z} + \boldsymbol{B}_z \boldsymbol{v}. \tag{8}$$

If we take input $\boldsymbol{v}$ as

$$v_i = y_{di}^{(r_i)} - \sum_{j=0}^{r_i-1} k_{i,j}(y_i^{(j)} - y_{di}^{(j)}), \tag{9}$$

where v_i, y_i, y_{di} are the ith element of $\boldsymbol{v}$, $\boldsymbol{y}$ and $\boldsymbol{y}_d$, respectively, and the superscript j of y_i and y_{di} in the parathesis denotes the j th derivative of them with respect to time t. We usually take joint angle $\boldsymbol{q}$ and/or joint stiffness $\boldsymbol{s}$ as $\boldsymbol{y}$. When we take $\boldsymbol{q}$ as $\boldsymbol{y}$, the joint angle, joint velocity, joint accelaration converge to the crresponding desired ones[12].

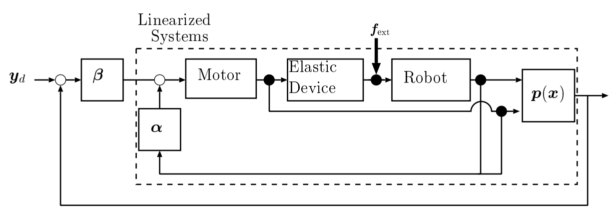

Figure 3: Non-linear Control Methods based on an exact feedback linearization

3.3 THE STRUCTURE OF ACM

Figure 4 show the block diagram of an adaptive controller. The controller adjusts its parameters dynamically with parameter update laws shown below. The input is given as follows:

$$\boldsymbol{\tau}_a = -\boldsymbol{A}_1 \boldsymbol{s}_\theta + \boldsymbol{J}_2^T \hat{\boldsymbol{f}} + \hat{\boldsymbol{\tau}}_a, \tag{10}$$

where $\boldsymbol{s}_\theta = \dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_d + \boldsymbol{A}_a(\boldsymbol{\theta} - \boldsymbol{\theta}_d)$ and $\boldsymbol{A}_1$ is the feedback gain. Both $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{\tau}}_a$ are the desired elastic force vector and the desired motor torque vector, respectively, and they can be written in regressor expression. And parameter update lows renew the estimated parameters of the regressors. The desired motor angle $\boldsymbol{\theta}_d$ is calculated with the estimated arm dynamics and the dynamics can be also written in regressor expression. For example, eq.(1) can be rewritten as follow:

$$\boldsymbol{Y}_m(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\phi}_m + \boldsymbol{J}_1(\boldsymbol{q})^T = \boldsymbol{f}_{\mathrm{ext}}, \tag{11}$$

where the first $\dot{\boldsymbol{q}}$ in $\boldsymbol{Y}_m$ represents the $\dot{\boldsymbol{q}}$ in the matrix $\boldsymbol{C}$ and the second $\dot{\boldsymbol{q}}$ the one multiplied by $\boldsymbol{C}$. Then the parameter update low of $\hat{\boldsymbol{\phi}}_m$ is given as follows:

$$\hat{\boldsymbol{\phi}}_m = \int_0^t \boldsymbol{\Gamma}_m \boldsymbol{Y}_m^T(\boldsymbol{q}(\tau), \dot{\boldsymbol{q}}(\tau), \dot{\boldsymbol{q}}_r(\tau), \ddot{\boldsymbol{q}}_r(\tau))\boldsymbol{s}_q d\tau \tag{12}$$

where $\dot{\boldsymbol{q}}_r = \dot{\boldsymbol{q}}_d - \boldsymbol{\Lambda}_m(\boldsymbol{q} - \boldsymbol{q}_d)$ and $\boldsymbol{s}_q = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_r$.

It is shown that the controller guarantees the convergence of the joint angle and joint velocity to desired ones[5].

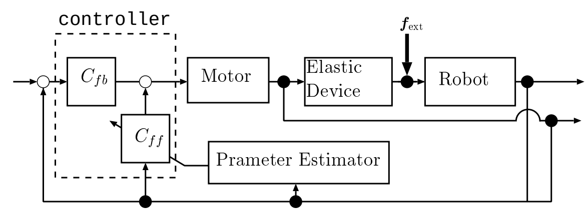

Figure 4: Adaptive Control Methods

3.4 THE STRUCTURE OF PDM

Figure 5 shows the block diagram of the PD controller based on motor angles. Note that this system does not include elastic devices in the control loop. The input is given as follows:

$$\boldsymbol{\tau}_a = -\boldsymbol{K}_p(\boldsymbol{\theta} - \boldsymbol{\theta}_d) - \boldsymbol{K}_v \dot{\boldsymbol{\theta}} + \boldsymbol{J}_2^T \hat{\boldsymbol{f}}. \tag{13}$$

The desire motor angle vector $\boldsymbol{\theta}_d$ and the feedforward force vector $\hat{\boldsymbol{f}}$ are determined taking the elasticity and the gravity force into account. This controller guanrantees the convergence to the desired joint angle[1].

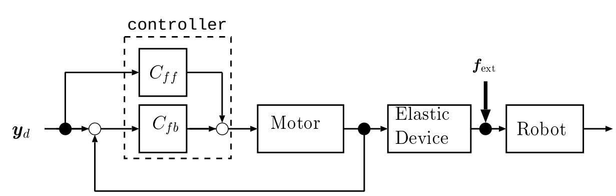

Figure 5: PD Control Methods based on motor angles

4 BEHAVIORS OF ELASTIC JOINT ROBOTS AGAINST DISTURBANCES

We give the disturbances to the elastic joint robots and investigate the behaviors of the elastic joint robot with the various controllers. We assume that the elastic robots have no force senser and no force controller but they have sufficient back-drivability to react on the disturbance.

4.1 IN THE CASE OF CTM

In this case we can assume that the dynamics of the inner servo loop can be neglected because of its fast convergence. We add $\boldsymbol{f}_{\text{ext}}$ to the right hand-side of eq.(3) and the substitution of eq.(4) into eq.(3) gives us the following equation:

$$\boldsymbol{M}(\boldsymbol{q})\{\Delta\ddot{\boldsymbol{q}} + \boldsymbol{K}_v\Delta\dot{\boldsymbol{q}} + \boldsymbol{K}_p\Delta\boldsymbol{q}\} = \boldsymbol{f}_{\text{ext}}. \tag{14}$$

As shown in **Figure 6**, the gain matrices of the input(4) gorvens the behavior of the robots and the mechanical elasticity is useless in this case.

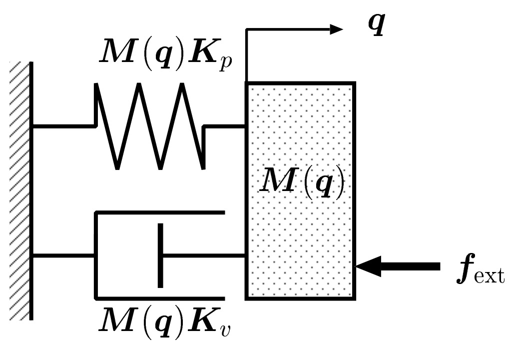

Figure 6: A steady-state model of an elastic joint robot driven by a computed torque controller

4.2 IN THE CASE OF NFL

When the external force $\boldsymbol{f}_{\text{ext}}$ affects on the robots, eq.(5) can be rewritten as follow:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_x(\boldsymbol{x}) + \boldsymbol{G}_x\boldsymbol{\tau}_a + \boldsymbol{G}_d\boldsymbol{f}_{\text{ext}}. \tag{15}$$

The substitution of eqs.(7) and (9) into eq.(15) yields the following:

$$\sum_{i=0}^{3}\boldsymbol{K}_{vi}(\boldsymbol{q}^{(i)} - \boldsymbol{q}_d^{(i)}) = \boldsymbol{X}(\boldsymbol{y})\boldsymbol{f}_{\text{ext}}, \tag{16}$$

where $\boldsymbol{K}_{v3}$ is the identity matrix, $\boldsymbol{X}(\boldsymbol{y})$ is a coefficient matrix that expresses effects of the coodinate transformation. The coodinate transformation is so complicated that the reaction also becomes very complicated.

4.3 IN THE CASE OF ACM

In the vincity of an equilibrium point the behaviors of the adaptive control systems is given as follow:

$$\boldsymbol{Y}_m(\boldsymbol{q}, \mathbf{o}, \mathbf{o}, \mathbf{o})\hat{\boldsymbol{\phi}}_m - \boldsymbol{f}_{\text{ext}} + \boldsymbol{K}\Delta\boldsymbol{q} = \mathbf{o}, \tag{17}$$

where $\boldsymbol{K}$ is the equivalant feedback gain that integrates the total feedback effects for the joints, the motors and the elastic forces. The first term of eq.(17)

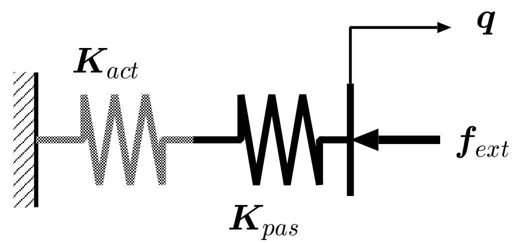

Figure 7: A steady-state model of an elastic joint robot driven by PD controller

has an estimate parameter $\hat{\boldsymbol{\phi}}_m$ with an integrater as shown in eq.(12). Thus the integrater absorbs the effect of the disturbances so that the trajecties converge to the desired ones.

4.4 IN THE CASE OF PDM

The behaviors of the robot in the vincity of an equilibrium point are gorvened the following:

$$\boldsymbol{f}_{\text{ext}} = \left\{\boldsymbol{K}_{pas}^{-1} + \boldsymbol{K}_{act}^{-1}\right\}^{-1}\Delta\boldsymbol{q}, \tag{18}$$

where $\boldsymbol{K}_{pas}$ is the mecanical joint stiffness $\boldsymbol{J}_1^T\boldsymbol{K}_t\boldsymbol{J}_1$ and $\boldsymbol{K}_{act}$ is the active feedback gain $\boldsymbol{J}_1^T\boldsymbol{J}_2^{-T}\boldsymbol{A}_1\boldsymbol{J}_2^{-1}\boldsymbol{J}_1$. As shown in **Figure 7** the robot behaves just same as the series springs of $\boldsymbol{K}_{\text{pas}}$ and $\boldsymbol{K}_{\text{act}}$. Thus the mechanical joint stiffness $\boldsymbol{K}_{pas}$ gorvens the behaviors of the robots when the motor has no back-drivability or we take $\boldsymbol{A}$ enough large.

5 SIMULATION RESULTS

In this section we will show some simulation results for a one D.O.F. tendon-driven robot with two elastic tendons, shown in **Figure 8**. We assume that the spring coeffcient of the tendons is linear. Thus the equation of motions of the robot can be expressed as:

$$\begin{aligned} M\ddot{q} + B\dot{q} + \boldsymbol{J}_1^T\boldsymbol{K}_t(\boldsymbol{J}_1 q + \boldsymbol{J}_2\boldsymbol{\theta}) &= f_{\text{ext}} \\ \boldsymbol{M}_a\ddot{\boldsymbol{\theta}} + \boldsymbol{B}_a\dot{\boldsymbol{\theta}} + \boldsymbol{J}_2\boldsymbol{K}_t(\boldsymbol{J}_1 q + \boldsymbol{J}_2\boldsymbol{\theta}) &= \boldsymbol{\tau}_a, \end{aligned} \tag{19}$$

where $\boldsymbol{J}_1 = (-r\ r)^T$ and r is the pulley radius mounted on the joint axis and is set to $0.02[m]$. $\boldsymbol{J}_2$ is the diagonal matrix whose diagonal elements are equal to $0.01[m]$. $\boldsymbol{K}_t$ is the spring coefficient matrix of the tendons. So the effective joint sfiffness s is given as follow:

$$s = \boldsymbol{J}_1^T\boldsymbol{K}_t\boldsymbol{J}_1. \tag{20}$$

The desired joint angle is set to $0[rad]$. Af first the robot is at the equiliburium and $-0.1[Nm/rad]$ of the step disturbance is given at one second.

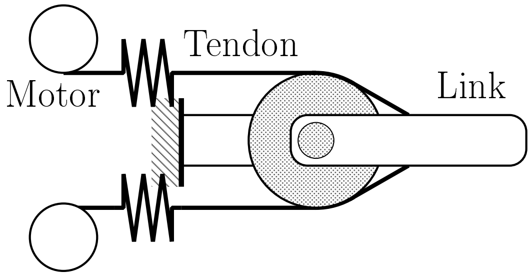

Figure 8: One D.O.F. tendon-driven robot with two elastic tendons

5.1 IN THE CASE OF CTM

Figure 9 shows the responces of the robot when it has the different values of joint stiffness. The stiffness is set to $10[Nm/rad]$ for the upper line, $7[Nm/rad]$ for the middle line and $3[Nm/rad]$ for the lower line. The effect of the joint stiffness is appeared in the transient responce, but is not appeared in the steady one.

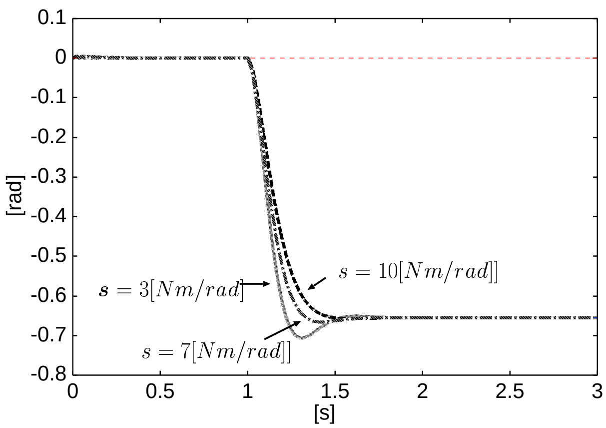

Figure 9: Step response of the elastic joint robot driven by CTM.

5.2 IN THE CASE OF NFL

Figure 10 show the responces of the robot driven by FLM with two kinds of the joint stiffness. The stiffness is set to $15[Nm/rad]$ for the dotted line and $10[Nm/rad]$ for the solid line. The shape of these line is quite similar but the steady state is quite different. This is the influence of the coodinate transformation.

5.3 IN THE CASE OF ACM

Figure 11 shows the step responce of the robot driven by ACM. In this case as we do not know the exact stiffness, the stiffness must be estimated the other parameter is done. Thus we focus on the steady responce only. When the disturbance is added to the robot, it

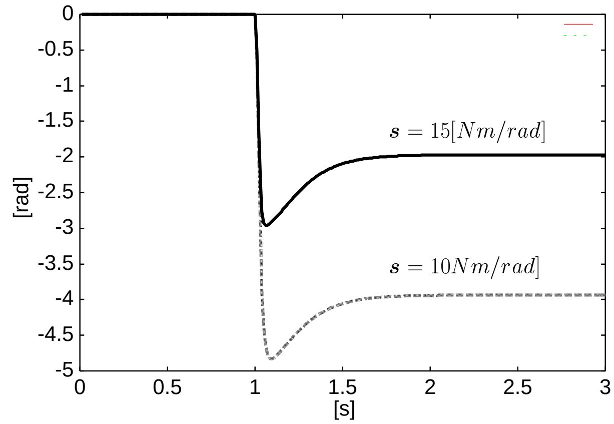

Figure 10: Step response of the elastic joint robot driven by NFL

reacts the disturbance at first. But the integral effect of the parameter update law absorbs the effect and it eliminates the position error.

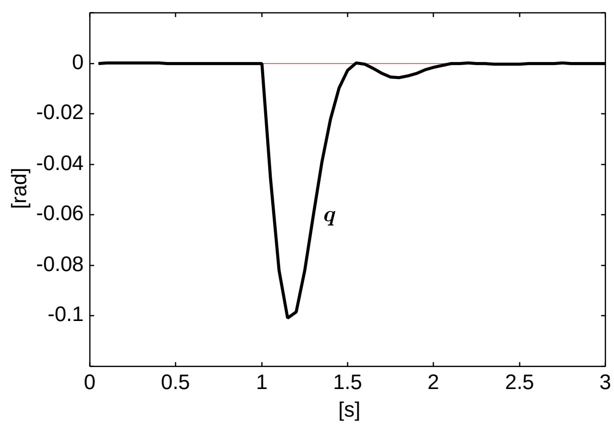

Figure 11: Step response of the elastic joint robot driven by Adaptive Control Method

5.4 IN THE CASE OF PDM

Figure 12 shows the responces of the robot with different feedback gains. The value of the position feedback gain K_p is set to $1.0[Nm/rad]$ for the solid line and $100.0[Nm/rad]$ for the dotted line, respectively. Thus their effective joint stiffness by the position feedback gains are $8[Nm/rad]$ and $800[Nm/rad]$, respectively. The joint stiffness of the robot is $7[Nm/rad]$ in both cases. In the case of the small feedback gain the mechanical joint stiffness cannot be neglect the effect of the gain so that the effective stiffness obeys eq.(18). In the case of the large gain, on the other hand, the mechanical stiffness governs the effective stiffness.

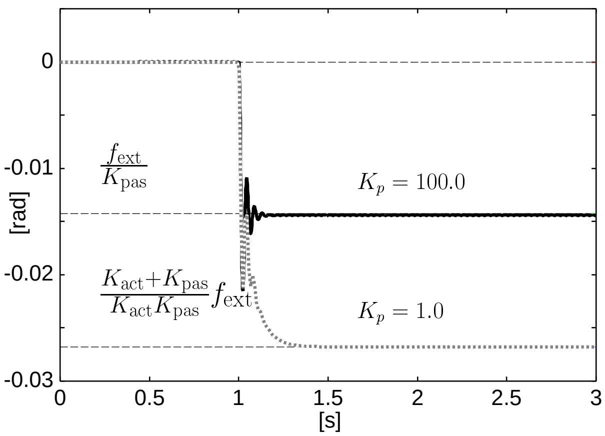

Figure 12: Step response of the elastic joint robot driven by PD Control Method

6 CONCLUSTION

In this paper we investigated behaviors of elastic joiont robots with mechanical elastic devices driven by various types of controllers against an external disturbance applied on the robots. Computed Torque Method perfectly gorverned the steady states by active compliance. But the mechanical compliance affected the transient responce. Adaptive Control Methods also gorverned the behavior actively, but the integrator in the parameter update law elminated the position errors. The mechanical compliance highly affected on the behaviors of the robot driven by Non-linear Feedback with exact Linearization. But the behaviors were almost unpredictable because of the complecity of the coordinate transformation. PD Control Method is the only controller of them for which the effect of mechanical compliance appears in the steady responce. So we can say that PD controller based on motor angles utilizes the mechanical compliance most effectively in four controllers.

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