係数体は、標数 2 とする。

(1) 新しく出てくる生成元の計算

$$
\begin{aligned}
a01 =& y^3 - x^{28}z \in [P]_{309}\\
b01 =& z^2 - x^{47}y \in [P]_{338}\\
c01 =& y^2z - x^{75} \in [P]_{375}\\
d02 =& (ya01b01 - c01^2)/x^{28}\\
=& yz^3 + x(\cdots) \in [P^{(2)}]_{610}\\
d03 =& (c01d02 - a01b01^2)/x^{19}\\
=& y^7z + x(\cdots) \in [P^{(3)}]_{890}\\
d04 =& (b01d03 - a01^2d02)/x^9\\
=& z^7 + x(\cdots) \in [P^{(4)}]_{1183}\\
d07 =& (d02^2d03 - a01^3d04)/x^9 \quad \text{NC}\\
=& y^2z^{11} + x(\cdots) \in [P^{(7)}]_{2065}\\
d09 =& (a01^2d07 - d02d03d04)/x\\
=& y^{26} + x(\cdots) \in [P^{(9)}]_{2678}\\
d15 =& (d03d04^3 - a01d07^2)/x^2\\
=& y^{43} + x(\cdots) \in [P^{(15)}]_{4429}\\
d25 =& (d02d03^3d07^2 - d04^4d09)/x^2\\
=& y^{62}z^6 + x(\cdots) \in [P^{(25)}]_{7400}\\
d29 =& (d02d04^3d15 - d03^2d07^2d09)/x^2\\
=& y^{80}z^2 + x(\cdots) \in [P^{(29)}]_{8578}\\
d32 =& (d03^3d04^2d15 - d07d25)/x\\
=& y^{82}z^6 + x(\cdots) \in [P^{(32)}]_{9460}\\
d37 =& (d03^5d07d15 - d04^2d29)/x\\
=& y^{98}z^5 + x(\cdots) \in [P^{(37)}]_{10939}\\
d41 =& (y^2a01^2d04d07^5 - zb01d02d03d07^5 - x^9yb01^2d04d07^5 - x^{10}yb01d04^6d07d09\\
& - x^{14}yd02^2d37 - x^{15}ya01^2d09d15^2 - x^{38}d04^8d09 - x^{39}d02d04^6d15 - x^{42}d04d37)/x^{43}\\
=& y^{116}z + x(\cdots) \in [P^{(41)}]_{12117}
\end{aligned}
$$

$$
\begin{aligned}
d43 =&(y^2a01^2d04d07^4d09 - zb01d02d03d07^4d09 - x^9yb01^2d04d07^4d09 - x^{10}za01d02d04d07^3d15\\
&- x^{11}ya01b01d02d07^2d25 - x^{12}ya01b01d02d07d32 - x^{13}yb01d02d03d37 - x^{14}ya01^4d09d15^2\\
&- x^{22}yd02d04d37 - x^{27}ya01d03d04d07^5 - x^{29}yd03d04d07^3d15 - x^{39}d02^2d07^2d25\\
&- x^{40}d02^2d07d32 - x^{41}a01^2d04d37 - x^{42}a01^2d41 - x^{45}d02d04^5d07^3 - x^{46}d02d03^2d07^5\\
&- x^{47}d04^7d15 - x^{48}d03^2d04^2d07^2d15)/x^{51}\\
=&y^{115}z^5 + x(\cdots) \in [P^{(43)}]_{12690}\\
d49 =&(yb01d04^4d07d25 - zd03^2d07^4d15 - xyb01d04^4d32 - x^4ya01^2d03^2d41 - x^7ya01^3d04d07^6\\
&- x^9ya01^2d04d07^4d15 - x^{13}yd03^2d43 - x^{16}yd07^7 - x^{18}b01^2d04d07^4d15\\
&- x^{19}b01d02d07^3d25 - x^{20}b01d02d07^2d32 - x^{21}a01d02^2d07d37 - x^{22}b01d02d03d43\\
&- x^{23}a01^4d15^3 - x^{28}d04^6d25 - x^{29}d04^5d29 - x^{30}d04^3d37 - x^{31}d02d04d43)/x^{32}\\
=&y^{134}z^4 + x(\cdots) \in [P^{(49)}]_{14478}
\end{aligned}
$$

mod x^{29} で計算

$$
\begin{aligned}
d53 =&(yb01d04^5d07d25 - zd03^2d04d07^4d15 - xyb01d04^5d32 - x^4yd02^2d49 - x^6yd02d04^{11}d07\\
&- x^7yb01d03d07^7 - x^{10}yd03d04d07^3d25 - x^{13}yd03^2d04d43 - x^{16}yd04d07^7\\
&- x^{18}b01^2d04^2d07^4d15 - x^{19}b01d02d04d07^3d25 - x^{20}b01d02d04d07^2d32\\
&- x^{21}a01b01d07^2d37 - x^{25}b01d04^6d07^4 - x^{28}d04^7d25 - x^{29}d04^6d29 - x^{32}d04d49)/x^{33}\\
=&y^{152} + x(\cdots) \in [P^{(53)}]_{15656}\\
d56 =&(yb01d04^4d07^2d25 - zd03^2d07^5d15 - xyb01d04^4d07d32 - x^4ya01d02d04d49\\
&- x^5ya01d02d53 - x^9ya01^2d04d07^5d15 - x^{13}yd03^2d07d43 - x^{16}yd07^8\\
&- x^{18}b01^2d04d07^5d15 - x^{19}b01d02d07^4d25 - x^{20}b01d02d07^3d32 - x^{21}a01d02^2d07^2d37\\
&- x^{22}b01d02d03d07d43 - x^{23}a01^3d04d49 - x^{24}a01^3d53 - x^{28}d04^6d07d25 - x^{29}d04^6d32)/x^{33}\\
=&y^{154}z^4 + x(\cdots) \in [P^{(56)}]_{16538}
\end{aligned}
$$

mod x^{29} で計算すればよい

ϕ

mod x^{34} で計算すればよい

mod x^{10} で計算すればよい

(2) $P^{(59)}$ の生成元

$$I := \text{下の 105 個の元で生成される } S \text{ のイデアル}$$

$B01^3D04^{14}$,	$B01D02D04^{14}$,	$D04^{13}D07$,	$B01D04^{11}D07^2$,	$B01^2D04^9D07^3$,
$D02D04^9D07^3$,	$B01D02D04^7D07^4$,	$D04^6D07^5$,	$B01D04^4D07^6$,	$B01^2D04^2D07^7$,
$D02D04^2D07^7$,	$B01D02D07^8$,	$C01D02D07^8$,	$A01D02D07^8$,	$B01A01^2D07^8$,
$D03D07^8$,	$A01^3D07^8$,	$D02D04^{12}D09$,	$D03^2D04D07^7$,	$D04^9D07^2D09$,
$B01D04^7D07^3D09$,	$D03^3D04^2D07^6$,	$D02D04^5D07^4D09$,	$B01D02D04^3D07^5D09$,	$D04^2D07^6D09$,
$B01D07^7D09$,	$C01D07^7D09$,	$D04^{11}D15$,	$B01D04^9D07D15$,	$B01^2D04^7D07^2D15$,
$D02D04^7D07^2D15$,	$B01D02D04^5D07^3D15$,	$D04^4D07^4D15$,	$B01D04^2D07^5D15$,	$B01^2D07^6D15$,
$D02D07^6D15$,	$A01B01D07^6D15$,	$A01C01D07^6D15$,	$A01^2D07^6D15$,	$D02D04^8D25$,
$B01D02D04^6D07D25$,	$D04^5D07^2D25$,	$B01D04^3D07^3D25$,	$B01^2D04D07^4D25$,	$D02D04D07^4D25$,
$A01B01D04D07^4D25$,	$D03^3D07^5D15$,	$A01^2D04D07^4D25$,	$B01D02D03D07^4D25$,	$D07^5D09D15$,
$D02D04^7D29$,	$B01D02D04^6D32$,	$D04^5D07D32$,	$B01D04^3D07^2D32$,	$B01^2D04D07^3D32$,
$D02D04D07^3D32$,	$A01B01D04D07^3D32$,	$D04^2D07^3D15^2$,	$A01^2D04D07^3D32$,	$B01D02D03D07^3D32$,
$A01D07^4D15^2$,	$D02D04^5D37$,	$B01D02D04^3D07D37$,	$D04^2D07^2D37$,	$B01D07^3D37$,
$C01D07^3D37$,	$A01D07^3D37$,	$B01D03D04D07^2D37$,	$C01D03D04D07^2D37$,	$A01D03D04D07^2D37$,
$A01^4D04D07^2D37$,	$D04^4D43$,	$B01D04^2D07D43$,	$B01^2D07^2D43$,	$D02D07^2D43$,
$A01B01D07^2D43$,	$A01C01D07^2D43$,	$A01^2D07^2D43$,	$A01B01D03D04D07D43$,	$A01C01D03D04D07D43$,
$A01^2D03D04D07D43$,	$A01^5D04D07D43$,	$D07^2D15^3$,	$D02D04^2D49$,	$B01D02D07D49$,
$D03^3D07D43$,	$A01D02D07D49$,	$A01^2B01D07D49$,	$D03D07D49$,	$A01^3D07D49$,
$A01^2B01D03D04D49$,	$D03^2D04D49$,	$A01^3D03D04D49$,	$B01^2D04D53$,	$D02D04D53$,
$B01D02D56$,	$C01D02D56$,	$A01D02D56$,	$A01^2B01D56$,	$D03D56$,
$A01^3D56$,	$A01^2B01D03D53$,	$D03^2D53$,	$A01^3D03D53$,	$A01^6D53$

I は $S = k[x, y, z]$ の斉次イデアルで $I \subset P^{(59)}$ を満たす。

$$L := \begin{pmatrix}
z^{104}, & yz^{103}, & y^2z^{102}, & y^4z^{101}, & y^6z^{100}, \\
y^7z^{99}, & y^9z^{98}, & y^{10}z^{97} & y^{12}z^{96}, & y^{14}z^{95}, \\
y^{15}z^{94}, & y^{17}z^{93}, & y^{19}z^{92}, & y^{20}z^{91}, & y^{22}z^{90}, \\
y^{23}z^{89}, & y^{25}z^{88}, & y^{27}z^{87}, & y^{28}z^{86}, & y^{30}z^{85}, \\
y^{32}z^{84}, & y^{33}z^{83}, & y^{35}z^{82}, & y^{37}z^{81}, & y^{38}z^{80}, \\
y^{40}z^{79}, & y^{42}z^{78}, & y^{43}z^{77}, & y^{45}z^{76}, & y^{47}z^{75}, \\
y^{48}z^{74}, & y^{50}z^{73}, & y^{51}z^{72}, & y^{53}z^{71}, & y^{55}z^{70}, \\
y^{56}z^{69}, & y^{58}z^{68}, & y^{60}z^{67}, & y^{61}z^{66}, & y^{63}z^{65}, \\
y^{65}z^{64}, & y^{66}z^{63}, & y^{68}z^{62}, & y^{70}z^{61}, & y^{71}z^{60}, \\
y^{73}z^{59}, & y^{74}z^{58}, & y^{76}z^{57}, & y^{78}z^{56}, & y^{79}z^{55}, \\
y^{81}z^{54}, & y^{83}z^{53}, & y^{84}z^{52}, & y^{86}z^{51}, & y^{88}z^{50}, \\
y^{89}z^{49}, & y^{91}z^{48}, & y^{92}z^{47}, & y^{94}z^{46}, & y^{96}z^{45}, \\
y^{97}z^{44}, & y^{99}z^{43}, & y^{101}z^{42}, & y^{102}z^{41}, & y^{104}z^{40}, \\
y^{106}z^{39}, & y^{107}z^{38}, & y^{109}z^{37}, & y^{111}z^{36}, & y^{112}z^{35}, \\
y^{114}z^{34}, & y^{115}z^{33}, & y^{117}z^{32}, & y^{119}z^{31}, & y^{120}z^{30}, \\
y^{122}z^{29}, & y^{124}z^{28}, & y^{125}z^{27}, & y^{127}z^{26}, & y^{129}z^{25}, \\
y^{130}z^{24}, & y^{132}z^{23}, & y^{133}z^{22}, & y^{135}z^{21}, & y^{137}z^{20}, \\
y^{138}z^{19}, & y^{140}z^{18}, & y^{142}z^{17}, & y^{143}z^{16}, & y^{145}z^{15}, \\
y^{147}z^{14}, & y^{148}z^{13}, & y^{150}z^{12}, & y^{152}z^{11}, & y^{153}z^{10}, \\
y^{155}z^{9}, & y^{157}z^{8}, & y^{158}z^{7}, & y^{160}z^{6}, & y^{161}z^{5}, \\
y^{163}z^{4}, & y^{165}z^{3}, & y^{166}z^{2}, & y^{168}z, & y^{170}
\end{pmatrix}$$

L は $k[y, z]$ のモノミアルイデアルで $(L, x)S = I + xS$ を満たす。

(3) colength の計算

$$\ell(S/I{+}xS) = \ell(k[y,z]/L) = 8850 = \left(\frac{59\times 60}{2}\right)\times 5 = e((x), S/P^{(59)}) = \ell(S/P^{(59)}{+}xS)$$

が言えて (二つ目の = は計算が必要)、$I = P^{(59)}$ がわかる。($P^{(59)} + xS = I + xS$ により $P^{(59)} = I + (xS \cap P^{(59)}) = I + xP^{(59)}$ となり、NAK によって $I = P^{(59)}$ とわかる。)

I の 105 個の生成元で、次数が最小なものは $D03\ D07^8$ であり、この次数は 17410 である。よって、

$$[P^{(59)}]_{17407} = 0$$

がわかった。