Transformation Groups and Hsiangs' Conviction after 46 years

Krzysztof Pawałowski (UAM Poznań, Poland)

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Talk on Friday, November 14, 2014, 14:30–15:30

For the $|A_5|$-Professors

Mikiya Masuda
Masaharu Morimoto
Kohhei Yamaguchi

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Proc. Amer. Math. Soc. 103 (1988) 1209–1212

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The even dimensional spheres S^4, S^8, S^{10}, S^{12}, S^{14}, S^{16}, ... are stably parallelizable manifolds

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Actions on finite CW complexes and disks

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The case of stably parallelizable fixed point sets

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$\mathbb{Z}$-cyclic manifolds as fixed point sets

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Homology spheres

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We may always assume that at a chosen point $x \in \Sigma^n$, the normal G-module is the realification of a complex G-module.

Stably parallelizable manifolds as fixed point sets

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Let G be a compact Lie group such that

(i) *G is a torus $S^1 \times \cdots \times S^1$, or*

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If we drop the assumtion that F is stably parallelizable, the same result is true for G as in (i), and for G as in (ii), we have to claim that "*F is $\mathbb{Z}_p$-acyclic and stably complex*" to prove that a similar statement for actions of G is true.

Stably parallelizable manifolds as fixed point sets

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Theorem C

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Let F be a stably parallelizable smooth manifold with $\partial F = \varnothing$. Then there exists a smooth action of G on some Euclidean space such that the fixed point set is diffeomorphic to F.

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Let F be a stably parallelizable closed smooth manifold. Then there exists a smooth action of G on a disk, resp. sphere, resp. complex projective spaces, such that a part of the fixed point set is diffeomorphic to F.

Stably parallelizable manifolds as fixed point sets

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R. Oliver, Comment. Math. Helv. 50 (1975) 155–177

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J. Symplectic Geom., Vol. 10, No. 1 (2012), 17–26.

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Actions on complex projective spaces

Y. Sato, Osaka J. Math. 28 (1991) 243–253

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Sato's Lemma

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For any compact Lie group G, there exists a smooth action of G on a complex projective space such that the fixed point set F is diffeomorphic to $\mathbb{C}P^2 \# \Sigma^4$, where Σ^4 is the homology sphere from Sato's Lemma.

Restrictions on F and actions on CW complexes

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Bob Oliver, Topology 35 (1996) 583–615

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- *The Euler characteristic $\chi(F) \equiv 1 \pmod{n_G}$ and the class $[\tau_F \oplus \nu]$ lies in the kernel of the map*

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- *There is a finite contractible G-CW complex X such that $X^G = F$ and $[\tau_F \oplus \nu]$ lies in the image of the restriction map*

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K. Pa., Topology 28 (1989) 273–289

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Corollary

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Let G be a finite group not of prime power order. Let F be a compact smooth manifold. Let ν be a G-vector bundle over F with $\dim \nu^G = 0$.

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Let G be a finite group not of prime power order. Let F be a compact smooth manifold. Let ν be a G-vector bundle over F with $\dim \nu^G = 0$. Then the two conditions are equivalent.

Restrictions on F and actions on disks

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Let G be a finite group not of prime power order. Let F be a compact smooth manifold. Let ν be a G-vector bundle over F with $\dim \nu^G = 0$. Then the two conditions are equivalent.

- *The Euler characteristic $\chi(F) \equiv 1 \pmod{n_G}$ and the class $[\tau_F \oplus \nu]$ lies in the kernel of the map*

$$\widetilde{KO}_G(F) \to \widetilde{KO}(F) \oplus \bigoplus_{P \in \mathcal{P}(G)} \widetilde{KO}_P(F)_{(p)}.$$

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ありがとうございました

For the $|A_5|$-Professors

Mikiya Masuda
Masaharu Morimoto
Kohhei Yamaguchi

御誕生日おめでとう ございます